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Essays in Applied Game Theory

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Contents

Introduction

This Thesis is a collection of three essays on applications of game theory to contexts such as licence and patents and, in general, to cultural products and cultural industries.

The first paper develops a model which tries to analyse incentives of corporate donations to open source. Opens source software (OSS) is developed by firms and individuals, and distributed for free. However, these contributions can hardly be explained by the usual considerations, such as the well known *warm glow*.

In the paper we develop the idea that companies, instead, may use open source software (OSS) as a strategic variable, in a market in which hardware and software are complements. Hardware firms may be willing to develop OSS in order to decrease the market power of the software producer, in order to charge a greater mark–up.

Moreover, due to the characteristics of information and public good of OSS, it is very interesting to study the welfare effects of strategic interactions and public intervention to support OSS.

Results are several, positive (private) contributions are possible; although, they are not socially optimal. OSS availability has a positive impact on hardware firms' profits and prices, and on social welfare. Software monopolist's profits and price decrease when OSS is available. The effect on the price of the hardware-closed source bundle depends on model parameters: when demand own-price elasticity is relatively high, it increases with respect to the case in which OSS is not developed.

In the second paper, we develop a similar topic by further consider the field of intellectual property. In our vision we think that incomplete information may affect strategic interactions in a system in which patent assessment is not perfectly reliable.

In this model we try to define patent complexity as the degree of difficulty in properly understanding the boundaries (breath) of a patent. Patent complexity determines the degree of spillovers of information released to the potential entrant. We build a Bayesian game and try to understand if firms can exploit asymmetric information and patent complexity to avoid entry and for other competitive behaviours.

Results are several. Entry deterrence is possible for certain values of parameters. There is an incentive in increasing the complexity, although, there exists an upper-bound which is inefficient, for the incumbent, to cross; this upper–bound is determined by the credibility of the strategic threat.

In the last paper we continue using a Bayesian approach to derive a game theoretic model. We investigate the selection of artists by a gallery with adverse selection and moral hazard and derive an optimal mechanism for cultural industries.

The model accounts for the possibility that artists' output is not homogeneous. Moreover, we think that being recognised as innovative by a system of gate-keepers could affect the price as well. These two characteristics create a market power, which can be exploited by galleries.

We study the relationship between innovation and productivity/creativity in the artist job market, as well as for the effect of *gate–keepers* in the art market. In a Bayesian-Nash equilibrium, the choice of the Gallery will depend on the artists' characteristics, as well on the gallery's market power. Furthermore, we find that a segmented market with gate-keeping, where some artists have no opportunity to bid to join a top gallery, has a negative impact on innovation.

Chapter 1

A Model of corporate donations to open source under hardware–software complementarity

Abstract

In recent years there has been an increasing diffusion of open source projects, as well as an increasing interest among scholars on the topic. Open source software (OSS) is developed by communities of programmers and users, usually sponsored by private firms; OSS is available in the public domain and redistributed for free.

In this paper a model of open and closed source software (CSS) competition will be presented. Hardware and software are complement goods and OSS is financed by hardware firms. There is a differentiated oligopoly of hardware–software bundles, in which firms compete in prices. Results are several; positive (hardware firm) contributions are possible, although, they are not socially optimal. OSS availability has a positive impact on social welfare, and on hardware firms' profits and prices. CSS firm's price and profits decrease when OSS is available. The effect on the price of the hardware–CSS bundle depends on demand own–price elasticity.

The model can explain the increasing participation in open source projects of embedded device producers. Hardware firms' incentives to contribute to OSS development process are greater when there is a relatively intensive competition among producers. Hardware firms use OSS to decrease the software monopolist's market power.

JEL Classification: L17; D21; D43; L11;

1 Introduction

In recent years there has been an increasing diffusion of open source projects (Lerner and Tirole, 2002), as well as an increasing interest among scholars on the topic. Open source software (OSS) is released under a special licence which does not put any restriction on the redistribution of the software and does not require any price, royalty or fee for the use or the redistribution (Rossi, 2005; Spiller and Wichmann, 2002).

Open source (OS) projects are developed by communities of programmers and users, usually sponsored by private firms and individuals; software is available in the public domain and redistributed for free. For an interesting analysis on OSS phenomenon see Spiller and Wichmann (2002).

The aim of the paper is threefold. First, the paper analyses competition between open and closed source software, in a market where hardware and software are complements. Second, motivations which determine positive contributions by hardware firms are considered, as well as, conditions for the existence of corporate contributions.

Finally, social welfare analysis will be carried out, to understand the impact of free software on welfare, and to asset the efficiency of public intervention instruments, such as transfers to open source and taxation to raise public revenues.

OS software is developed independently by a software foundation, which is financed by hardware firms. Exogenous and public donations are also considered. The model is a two stage game with perfect information. In the first stage hardware firms decide the amount of contributions to the OS foundation. These contributions will finance the OSS R&D process. With a certain probability – increasing in the amount of contributions – OSS development will be successful. In second stage, price competition takes place in a differentiated market with hardware and software bundles. Marginal costs, for both hardware and software, are normalised to zero. The basic setting is that of Economides and Salop (1992).

The key consideration is that hardware and software are complement goods. The presence of a single monopolist in the software market has a negative effect on hardware firms' profits. Therefore, vertical and horizontal externalities must be considered (Economides and Salop, 1992). Through OSS, hardware firms may increase their prices and profits.

For its characteristics, the model can explain OSS contributions by developers of embedded devices (such as smartphones, tablets, etc.). The relatively small price of these devices, and the complementarity between devices and their operating systems, constitutes an incentive to finance the development of an open source operating system[1].

Results are several, positive (private) contributions are possible; although, they are not socially optimal. OSS availability has a positive impact on hardware firms' profits and prices, and on social welfare. CSS firm's profits and price decrease when OSS is available. The effect on the hardware-CSS bundle's price depends on model parameters; when demand own–price elasticity is relatively high, the price of CSS–based bundles increases with respect to the case in which OSS is not developped.

[1]Examples of embedded devices' operating systems, which are open source, are Android by Google, Maemo/MeeGo by Nokia–Intel, SymbianOS by Nokia, Sony Ericsson and Motorola. For further information see Dorokhova et al. (2009), Anvaari and Jansen (2010) and Lin and Ye (2009).

The paper is organised as follows. In next section we present the literature background of the paper. Then we develop the model and show theoretical results. Social welfare analysis and instruments for public intervention will follow. Concluding remarks will be presented in the last section.

2 Related literature

Due to the characteristic of public good of OSS, a great deal of attention has been put on motivations and incentives of open source project contributors. The analysis of incentives, however, must take into account heterogeneity of developers, contributors and users (Rossi, 2005), which could lead to a variety of coexisting motivations.

With regards to the literature, incentives are usually divided in extrinsic and intrinsic motivations[2] (Rossi, 2005; Krishnamurthy, 2006). In the former group, contributions determine present or future external benefits. While motivations in the second group may be associated to a *per se* pleasure in contributing, such as, for instance, the well known *"warm glow"* effect (Andreoni, 1990). Moreover, motivation analysis must take into account the differences between single individuals and firms.

Lerner and Tirole (2002) focus their attention on programmers' motivations. They argue that users have three main motivations linked to (1) the need of solving a problem they face (such as program bugs), (2) to benefits from signalling their skills in the job market, and (3) to benefits from peer recognition.

However, some remarks should be moved to these motivations. Reputation incen-

[2]On the topic, see Lakhani and Wolf (2005); Hars and Ou (2002); Lakhani and von Hippel (2003); Lerner and Tirole (2005).

tives, for instance, do not explain the creation of new projects and some activities – for instance, bug reporting, creation and/or translation of software documentation, etc. – which are done by the wide majority of OSS contributors (Rossi, 2005).

Krishnamurthy (2006) presents an interesting review of surveys studying motivations in OSS development. Heterogeneity of developers determines a different rank of motivations among them. However, he finds four major factors which influence contributions: financial incentives, nature of task, group size and group structure.

With regards to firm's motivations, according to Wichmann (2002), firms have different positions towards OS project, which depend both on the type of software developed and on the core business of the firm. These motivations may be collocated in four major groups. The first reason may be found on the need of system standardisation. Since OSS is freely available and it can be modified by everyone, it is a good candidate as a common standard[3]. Furthermore, OS software can be used as a low cost component in bundles, firms can – consequently – use it to increase revenues. Strategic behaviours, such as competition with a dominant firm, and willingness to enable compatibility between their products and the available software, are the other two incentives.

We think that corporate contributions in the operating system market may be explained by the previous incentives (low cost component in bundles, competition with a dominant firm). Hardware firms can use OSS as substitute of the proprietary software to increase their profits.

[3]The problem can be referred to the literature which analyses the adoption of different standards. Prisoners' Dilemma problems, as well as path dependency, may arise in this context. An open standard may solve these problems.

Two branches of literature must be considered as background for this paper. On the one hand, since hardware and software are complement goods, we have to look at models of competition in complementary markets. Economides and Salop (1992) analyse competition when complement goods may be combined, deriving the equilibrium prices in the market. They also analyse horizontal and vertical integration and derive welfare properties. Similarly, Choi (2008) analyses the effects of mergers in markets with complements, when it is possible to sell bundles in the market. Welfare properties of mergers are not clear, they *"could entail both pro-competitive and anti-competitive effects"* (Choi, 2008).

On the other hand, competition between open source and closed source software must be analysed. Several authors examine open/closed source software competition: Bitzer (2004); Bitzer and Schroder (2007); Casadesus-Masanell and Ghemawat (2006); Dalle and Jullien (2003); Economides and Katsamakas (2006); Lanzi (2009); Mustonen (2003); Schmidt and Schnitzer (2003).

Economides and Katsamakas (2006) develop a model of competition in a two–sided market. Firms develop a pricing strategy which takes into account both direct users and other software firms, the latter produces complementary applications. Main results are that open source availability determines a greater software variety, and in a open/closed source competition *"the proprietary system most likely dominates both in terms of market share and profitability"* (Economides and Katsamakas, 2006).

Dalle and Jullien (2003) analyse competition between MS Windows and Linux in the server operating system market. The presence of network effects – due to compatibility issues between different operating systems – as well as *"strong positive local externalities due to the proselytism of Linux adopters"* (Dalle and Jullien, 2003) are fundamental factors which affect adoption of the new software (Linux). Under certain conditions, adoption of the new technology is fairly rapid.

Windows–Linux competition inspires also the model of Casadesus-Masanell and Ghemawat (2006). They develop a competition model between software houses, where one product has price equal to zero and cumulative output affects the relative position (due to network externalities). The model, however, is unable to explain the economic behaviour of open source software producers.

Mustonen (2003) analyses the competitive threat faced by a software monopolist. OSS is developed by programmers. Software prices and monopoly profits decrease – if some conditions are met – under open source software availability.

Schmidt and Schnitzer (2003) develop a model of spatial competition between OSS and CSS firms, where network effects are null, due to perfect compatibility between the two technologies. Users face transportation (adaptation) costs in a Hotelling fashion. Moreover, only a part of the user population can choose what software to adopt; other two groups of users will use either OS or CS software. Due to the *lock–in* effect, CSS price increases when the number of OSS users increases. Innovation incentives, instead, decrease when OSS users increase in number.

Different results are achieved removing the above assumptions. For instance, Bitzer (2004) develops a Launhardt-Hotelling model of duopoly competition between OSS and CSS. R&D costs to develop OSS are zero. Users heterogeneity, among the different hardware platforms, determines the amount of strategic pressure on CSS price. Moreover, *"the absence of development costs for the Linux firm may induce the incumbent to stop any further development of its operating system"* (Bitzer, 2004).

Bitzer and Schroder (2007), furthermore, analyse the impact of OSS on innovation, in a model where demand depends on technological content of software. Entry of a new OSS firm – from monopoly to duopoly – determines a higher technological level chosen. The result holds whether incumbent is a closed source firm or an open source one. Lower costs for OSS firms in a pure OS competition (*i.e.* in a duopoly between two OSS firms), as well as higher pay–off for the proprietary software company in a mixed OS–CS competition, determine higher innovation rate than, respectively, mixed competition and pure OS competition.

Lanzi (2009) sets up a two-stage model of quality and price decision, with *lock–in* effects, externalities due to quality, and perfect compatibility among software platforms. He considers also the accumulated experience of users, which can alter the market structure. He finds two major results. In a duopoly between closed and open source software, CSS price decreases (with respect to monopoly price) if CSS platform is bigger than OSS one and users are experienced. The same result holds if the CSS network is bigger than OSS one, but users have no experience. Under OSS availability the quality of software increases. Finally, given the experience accumulation equation, the ratio of OSS and CSS opportunity costs will define the market structure; which can be either a shared market, or a market where only either OSS or CSS software is available.

The aim of this paper is to create a theoretical background to explain corporate donations to OSS. Therefore we endogenise corporate motivations to contribute to

OSS in a framework which considers CSS–OSS competition.

Our contribution does not merely analyse competition between OSS and CSS and its market outcome, but (1) we extend the theoretical framework to include a complement good (hardware), (2) we consider motivations which lead firms to contribute to OSS projects, and (3) we also consider welfare implications and outline the effect of public policies to sustain OSS development.

At the best of our knowledge this paper is the first theoretical paper which considers corporate donations and complementarity between hardware and software. Moreover, we find this framework to be consistent with the development of open source operating systems for embedded devices (for instance, Android or MeeGo).

3 The model

This paper puts a great deal of attention on companies' contribution to open source. Complementarity among hardware and software, could lead hardware companies to contribute to open source development.

The basic setting is derived by Economides and Salop (1992) and Choi (2008). There are two hardware firms and one (closed source) software monopolist, and an OSS foundation. The combination of different brands of hardware and typologies of software creates four different composite products, which from now on will be referred as personal computer systems (PCS). Therefore, there are four PCS denoted as H_1CS, H_2CS, H_1OS and H_2OS, with regards to hardware (H_1 and H_2) and software components (Open and Closed source).

Composite goods (PCS) are sold in a differentiated oligopoly. Hardware and software are fully compatible. The demand system is assumed linear and symmetric, and is equal to the following expressions:

$$\begin{aligned} q_{H_1CS} &= a - bp_{H_1CS} + cp_{H_2CS} + cp_{H_1OS} + cp_{H_2OS} \\ q_{H_2CS} &= a - bp_{H_2CS} + cp_{H_1CS} + cp_{H_1OS} + cp_{H_2OS} \\ q_{H_1OS} &= a - bp_{H_1OS} + cp_{H_2CS} + cp_{H_1CS} + cp_{H_2OS} \\ q_{H_1OS} &= a - bp_{H_2OS} + cp_{H_2CS} + cp_{H_1OS} + cp_{H_1CS} \end{aligned} \tag{3.1}$$

Where p_{H_1CS} is the generic price of the PCS composed by hardware produced by firm H_1 and the closed source software. Trivially this price is the sum of the two prices. Due to the nature of open source, however, $p_{OS} = 0$. Consumers' choice can be interpreted as the decision of buying a bundle of hardware and CS software or buying only a hardware device and use the freely available software. Consequently, the demand system could be rewritten as in expression (3.2).

$$\begin{aligned} q_{H_1CS} &= a - (b-c)p_{H_1} - (b-c)p_{CS} + 2cp_{H_2} \\ q_{H_2CS} &= a - (b-c)p_{H_2} - (b-c)p_{CS} + 2cp_{H_1} \\ q_{H_1OS} &= a - (b-c)p_{H_1} + 2cp_{CS} + 2cp_{H_2} \\ q_{H_1OS} &= a - (b-c)p_{H_2} + 2cp_{CS} + 2cp_{H_1} \end{aligned} \tag{3.2}$$

Cellini et al. (2004) show that the equation system in expression (3.1) is the result of a particular quasi-linear individual utility function. Gross substitutability among all PCS is assumed by imposing $b > 3c$ (Choi, 2008). Marginal costs to produce hardware and software are normalised to zero.

Demand for firm i's products is the sum of all demands for composite goods which contain firm i's hardware (or software) (Economides and Salop, 1992; Choi, 2008). So, for instance, $q_{H_1} = q_{H_1CS} + q_{H_1OS}$ and $q_{CS} = q_{H_1CS} + q_{H_2CS}$.

The game is divided in two stages. Perfect information is assumed. In the first stage, there are two hardware firms and a software monopolist. Hardware firms can voluntary contribute to an Open Source Foundation. Companies' donations are used to finance the open source software R&D process, they affect the probability of OS software development and release. OSS is successfully developed with probability $P(F_{TOT})$; where $F_{TOT} = F_{H_1}+F_{H_2}+\bar{F}$. Parameters F_{H_1}, F_{H_2} and $\bar{F}$ are, respectively, hardware firm 1's donations, firm 2's donations and exogenous donations. F_{TOT} represents, therefore, total amount of donations to open source.

The probability $P(F_{TOT})$ is increasing in donations (*i.e.* $\partial P(F_{TOT})/\partial F > 0$), while we assume the second derivative to be negative. Therefore, we are assuming marginally decreasing returns of R&D investments.

In the second stage two outcomes are possible. With probability $P(F_{TOT})$ the open source software is developed and open source market exists. With opposite probability only closed source market is available. In the last stage price competition will take place, which depends on R&D outcomes.

4 Second stage optimisation

4.1 Absence of OS Software

When only CS software is available, the demand system in expression (3.1) should be modified to take into account the absence of two composite goods. This is of particular importance to analyse welfare properties of the two settings and voluntary contributions to the OS foundation, and to take into account substitution path towards other bundles, while maintaining the above system of equations.

When open source market is not released, quantities of OSS–based bundles are equal to zero:

$$q_{H_1OS} = q_{H_2OS} = 0 \tag{4.1}$$

Consequently, the demand system in expression (3.1) can be rewritten as[4]:

$$\begin{aligned} q_{H_1CS} &= \tfrac{a(b+c)}{b-c} - \tfrac{(b-2c)(b+c)}{b-c} p_{H_1CS} + \tfrac{c(b+c)}{b-c} p_{H_2CS} \\ &= \alpha - \beta p_{H_1CS} + \gamma p_{H_2CS} \\ q_{H_2CS} &= \tfrac{a(b+c)}{b-c} - \tfrac{(b-2c)(b+c)}{b-c} p_{H_2CS} + \tfrac{c(b+c)}{b-c} p_{H_1CS} \\ &= \alpha - \beta p_{H_2CS} + \gamma p_{H_1CS} \\ q_{H_1OS} &= q_{H_1OS} = 0 \end{aligned} \tag{4.2}$$

[4]Results are derived by imposing $q_{H_1OS} = q_{H_2OS} = 0$ in the system shown in Appendix E.

by imposing $\alpha = \frac{a(b+c)}{b-c}$, $\beta = \frac{(b-2c)(b+c)}{b-c}$ and $\gamma = \frac{c(b+c)}{b-c}$. It should be stressed that, in this way, substitution paths – towards bundles with CS software – are preserved. In this fashion, part of the demand for the two composite goods is substituted with demand for bundles with CS software. Furthermore, it can be easy to compare the two cases of absence and presence of open source software. As before the system in expression (4.2) can be expressed as follows:

$$\begin{aligned} q_{H_1CS} &= \alpha - \beta p_{H_1} + \gamma p_{H_2} - (\beta - \gamma) p_{CS} \\ q_{H_2CS} &= \alpha - \beta p_{H_2} + \gamma p_{H_1} - (\beta - \gamma) p_{CS} \end{aligned} \tag{4.3}$$

Property 1. *Parameters in the two systems of demands – in absence of OS software, and with OSS – are such that* $\alpha > a$*,* $\beta < b$*,* $\gamma > c$ *and* $\beta - \gamma < b - c$*.*

Proof. See Appendix A □

These results are quite intuitive. The absence of OS software leads to demands for bundles with CS software which have a greater intercept; own price derivative is smaller and cross-product derivative is bigger than the case of OSS availability. Demand is less sensitive to changes in the own price, while is more sensitive to changes in other prices. The absence of OSS has the effect of increase the oligopolistic power, but while competitions among hardware companies is more intense ($\gamma > c$), the CS company is a monopolist.

In this case, demands for hardware firms are $q_{Hi} = q_{HiCS}$ with $i = 1, 2$, while CS software company will face a demand equal to the sum of the two quantities.

The generic hardware firm i will maximise its profit function, given that OS software was unsuccessfully developed. Optimisation problem is:

$$\begin{aligned} &\max_{p_{Hi}} q_{HiCS} p_{Hi} \\ &\max_{p_{Hi}} \left[\alpha - \beta p_{Hi} + \gamma p_{Hj} - (\beta - \gamma) p_{CS}\right] p_{Hi}; \quad i, j = 1, 2; \quad i \neq j \end{aligned} \tag{4.4}$$

Whose first order condition leads to:

$$p_{Hi} = \frac{\alpha - (\beta - \gamma) p_{CS} + \gamma p_{Hj}}{2\beta}; \quad i, j = 1, 2; \quad i \neq j \tag{4.5}$$

Closed Source monopolist will maximise the following profit function:

$$\begin{aligned} &\max_{p_{CS}} \left[q_{H_1CS} + q_{H_2CS}\right] p_{CS} \\ &\max_{p_{CS}} \left[2\alpha - 2(\beta - \gamma) p_{CS} - (\beta - \gamma)(p_{H_1} + p_{H_2})\right] p_{CS}; \end{aligned} \tag{4.6}$$

First order condition can be expressed as:

$$p_{CS} = \frac{2\alpha - (\beta - \gamma)(p_{H_1} + p_{H_2})}{4(\beta - \gamma)}; \tag{4.7}$$

From the system of first order conditions, equilibrium prices and quantities are derived. Second order conditions are satisfied due to concavity of the profit functions.

Proposition 1. *In absence of open source market, price competition in second stage* *results in the following equilibrium prices, quantities and profits:*

$$p_{CS}^{nos} = \frac{\alpha\beta}{(\beta-\gamma)(3\beta-\gamma)}; \qquad q_{CS}^{nos} = 2\frac{\alpha\beta}{3\beta-\gamma}; \qquad \pi_{CS}^{nos} = 2\frac{\alpha^2\beta^2}{(\beta-\gamma)(3\beta-\gamma)^2}$$

$$p_{H_1}^{nos} = p_{H_2}^{nos} = \frac{\alpha}{(3\beta-\gamma)}; \quad q_{H_1}^{nos} = q_{H_2}^{nos} = \frac{\alpha\beta}{3\beta-\gamma}; \quad \pi_{H_1}^{nos} = \pi_{H_2}^{nos} = \frac{\alpha^2\beta}{(3\beta-\gamma)^2} \tag{4.8}$$

Proof. Equilibrium is derived from the system of first order conditions expressed in (4.5) and (4.7). Equilibrium may also be expressed using original parameters a, b and c (See Appendix B). □

4.2 Price competition under OS software availability

With probability $P(F_{TOT})$ the open source software is developed. Therefore, a generic hardware firm will maximise the following profit function:

$$\max_{p_{Hi}} [q_{HiCS} + q_{HiOS}]\, p_{Hi}$$

$$\max_{p_{Hi}} [2a - 2(b-c)p_{Hi} + 4cp_{Hj} - (b-3c)p_{CS}]\, p_{Hi}; \quad i,j = 1,2; \quad i \neq j \tag{4.9}$$

Whose first order condition is:

$$p_{Hi} = \frac{2a-(b-3c)p_{CS}+4cp_{Hj}}{4(b-c)}; \quad i,j = 1,2; \quad i \neq j \tag{4.10}$$

The maximisation problem for the closed source monopolist is:

$$\max_{p_{CS}} \left[q_{H_1CS} + q_{H_2CS}\right] p_{CS}$$

$$\max_{p_{CS}} \left[2a - 2(b-c)p_{CS} - (b-3c)(p_{H_1} + p_{H_2})\right] p_{CS}; \quad i, j = 1, 2; \quad i \neq j \tag{4.11}$$

Whose first order condition is:

$$p_{CS} = \frac{2a - (b-3c)(p_{H_1} + p_{H_2})}{4(b-c)}; \tag{4.12}$$

As before, equilibrium is derived from the system of first order conditions. Second order conditions are met due to concavity of objective functions.

Proposition 2. *Under open source software availability, equilibrium prices, quantities and profits are:*

$$p_{CS}^{os} = \frac{2a(b-c)}{(b-3c)(7b-5c)+8c(b-c)}; \qquad q_{CS}^{os} = \frac{4a(b-c)^2}{(b-3c)(7b-5c)+8c(b-c)}; \qquad \pi_{CS}^{os} = \frac{8a^2(b-c)^3}{((b-3c)(7b-5c)+8c(b-c))^2}$$

$$p_{H_1}^{os} = p_{H_2}^{os} = \frac{a(3b-c)}{(b-3c)(7b-5c)+8c(b-c)}; \quad q_{H_1}^{os} = q_{H_2}^{os} = \frac{2a(b-c)(3b-c)}{(b-3c)(7b-5c)+8c(b-c)}; \quad \pi_{H_1}^{os} = \pi_{H_2}^{os} = \frac{2a^2(b-c)(3b-c)^2}{((b-3c)(7b-5c)+8c(b-c))^2} \tag{4.13}$$

Proof. Equilibrium is derived from the system of first order conditions expressed in (4.10) and (4.12). See Appendix C for composite goods' equilibrium quantities. □

4.3 Comparative Statics

Property 2. *Under OS availability, hardware prices and profits increase and CS software price decreases,* i.e*:*

$$
\begin{aligned}
&p_{Hi}^{nos} < p_{Hi}^{os}; \qquad i = 1, 2 \\
&p_{CS}^{nos} > p_{CS}^{os}; \\
&\pi_{Hi}^{nos} < \pi_{Hi}^{os}; \qquad i = 1, 2 \\
&\pi_{CS}^{nos} > \pi_{CS}^{os};
\end{aligned} \tag{4.14}
$$

The effect on p_{HiCS} $(i = 1, 2)$ is not univocally determined. It will increase under OS availability if $b > 14.8214c$.

Proof. See Appendix D. □

Availability of free software leads to several changes in equilibrium prices and quantities. When OS software is developed, the CSS producer competes in a differentiated oligopoly with OS. Therefore, he looses market power. Hardware firms, consequently, may charge a bigger mark–up, leading to higher prices and profits for hardware firms.

OS availability increases hardware producers' market power, because of two reasons. On the one hand, there is an additional demand for hardware which can be exploited. Secondly, under OSS availability the CS monopolist has to compete in prices with the OSS foundation, the latter produces a software with price equal to zero.

When OS is available, a generic hardware firm faces two markets. In one ($HiOS$) the hardware firm behaves like an integrated firm which produces two complementary products (OSS is freely available, therefore the well-known Cournot problem of double mark-up does not exist). Consequently, $p^{os}_{HiOS} = p^{os}_{Hi}$ tends to increase with respect to p^{nos}_{Hi}. In the other market ($HiCS$), the CSS producer still charges a mark–up.

In the canonical Cournot problem – with two complementary monopolies – the effect on the overall price (p_{HiCS}) is unambiguous, *"joint ownership by a single integrated monopolist reduces the sum of the two prices, relative to the equilibrium prices of the independent monopolists"* (Economides and Salop, 1992). In this setting however, prices of PC systems with closed source software may increase. This because there is not joint ownership in the production of CSS-based devices, and also because hardware firms could increase their prices in order to exploit their greater market power under OSS availability.

When b is "big enough" ($b > 14.8214c$) we have that $p^{nos}_{HiCS} < p^{os}_{HiCS}$. Therefore, when the own price derivative is relatively big, the effect on hardware price (which increases under OS availability) outweighs the decrease of software price. The overall effect is an increase in the composite good's price.

The effect on firms' profits is unambiguous. Under OS availability, hardware firms increase their profits. The CS producer, on the contrary, is worse off when OSS

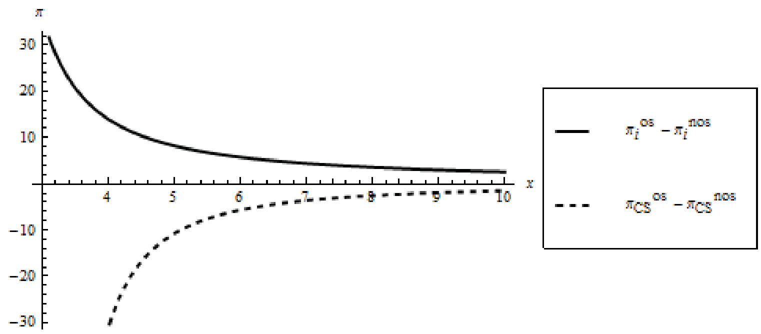

Figure 1: Difference in hardware and software firms' (second stage) profits with regards of OSS availability as functions of x, with $b = xc$, $x > 3$, $a = 5$ and $c = 0.35$

is realised. This can be seen in Figure n. 1, where the differences $\pi_{H_i}^{os} - \pi_{H_i}^{nos}$ and $\pi_{CS}^{os} - \pi_{CS}^{nos}$ are depicted as a function of x (where $b = xc$ and $x > 3$).

The new parameter x is a standardised (with respect to c) measure for the demand own price derivative, and it affects own price elasticity of demand. The smaller is x, the smaller is the demand own-price effect. This sensitivity measure is standardised w.r.t. c, and it represents the degree of differentiation of the PCS, goods are more homogeneous when x is smaller.

Trivially, the greater x the smaller are the differences in hardware firms profits between the two possible R&D outcomes (OSS successfully developed or not). On the contrary, the (negative) difference in the CSS firm's profits decreases with respect to x.

In Figure n. 2 we show the difference of the two industries profits in the two possible outcomes (*i.e.* $2\left(\pi_{H_i}^{os} - \pi_{H_i}^{nos}\right) + (\pi_{CS}^{os} - \pi_{CS}^{nos})$). As it can be seen, the relationship is not monotonic.

When x is relatively small – and, therefore, $b \to 3c$ – goods tend to be homo-

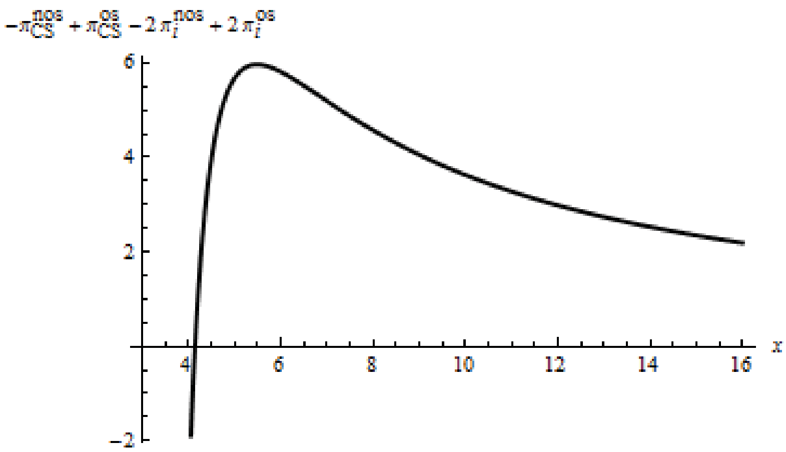

Figure 2: Difference the two industries (second stage) profits with regards of OSS availability as function of x, with $b = xc$, $x > 3$, $a = 5$ and $c = 0.35$

geneous. Therefore, the two hardware firms compete almost in a Bertrand fashion, while the CS monopolist has a relatively high market power. The presence of OS software (which has price equal to zero) has a huge effect on profits. The CSS producer, now, has to compete in prices with a virtual firm which commits to a price equal to zero. Then, hardware firms gain a great market power under OSS availability. The more is differentiated the market, the lower is the effect on profits, because losses (of the software monopolist) and gains (of hardware firms) under OSS availability decrease with x.

This would be consistent with the recent development of several open source projects for embedded devices. In the market for smartphones, tablets, etc. we can argue that the variable x is relatively small. This is the case because those devices are in some degree homogeneous: they have similar functionalities and characteristics both in hardware and software, they are mostly sold to consumers without specific needs (general public oriented)[5], and applications are available for several operating systems[6] (network effects are smaller than, for instance, in the personal computer market – e.g. Windows and Linux – because of smaller compatibility issues).

Competition among device producers may be relatively intense; with open source operating systems, producers may increase their mark-up and their profits[7].

5 First Stage decision

In the first stage, both hardware firms choose the amount of contributions they want to donate to the OS foundation. With probability $P(F_{TOT})$ open source software will be successfully developed, and will be freely available to consumers.

A generic hardware firm i maximises its expected profits decreased by the amount of contribution devolved to the OS foundation, *i.e.*:

$$F^*_{H_i} = \arg\max_{F_{Hi}} P(F_{TOT})\pi^{os}_{Hi} + (1 - P(F_{TOT}))\pi^{nos}_{Hi} - F_{Hi}; \qquad i = i, 2$$
$$F^*_{H_i} = \arg\max_{F_{Hi}} \pi^{nos}_{Hi} + P(F_{TOT})\left[\pi^{os}_{Hi} - \pi^{nos}_{Hi}\right] - F_{Hi}; \qquad i = i, 2 \tag{5.1}$$

The difference $\pi^{os}_{Hi} - \pi^{nos}_{Hi}$ is positive (Property 2) and is equal to expression (D.3).

[5]The absence of specific needs to be met could leads to a greater substitutability among devices

[6]The majority of the (the most downloaded) 'Apps' are available, for instance, both in the Apple Store and the Android (Google Play) Market

[7]Lin and Ye (2009), for instance, argue that *"the price of an OS has a direct impact on device makers' decisions. They tend to choose the OS with a better price p_i so that their devices can get a price advantage."*

The first order condition is:

$$\left[\pi_{Hi}^{os} - \pi_{Hi}^{nos}\right] \frac{\partial P(F_{TOT})}{\partial F_{Hi}}\Big|_{F_{Hi}=F_{Hi}^{*}} = 1; \qquad i = i, 2 \tag{5.2}$$

Second order condition is satisfied due to concavity of the probability function, and, consequently, of the profit function. Furthermore, to have a positive amount of contributions, the following condition must be satisfied:

$$\pi_{Hi}^{nos} + P(F_{H_1}^{*} + F_{H_2}^{*} + \bar{F})\left[\pi_{Hi}^{os} - \pi_{Hi}^{nos}\right] - F_{Hi}^{*} \geq \pi_{Hi}^{nos}; \qquad i = 1, 2 \tag{5.3}$$

According to this condition, hardware firms improve their (expected) profits by contributing to open source software. If this condition is not verified, a positive contribution will not be the market outcome.

6 Consumer Surplus

It is easy to show (See Appendix E) that, under OS availability, consumer surplus is:

$$CS^{os} = \frac{1}{2}\left[\sum_{\forall k}\left[\frac{a}{b-3c} - p_k\right] q_k\right] \tag{6.1}$$

for $k = \{H_1CS, H_2CS, H_1OS, H_2OS\}$.

While, when OS is not developed only two markets exist, and consumer surplus results in the following expression:

$$CS^{nos} = \frac{1}{2}\left[\sum_{\forall k}\left[\frac{\alpha}{\beta-\gamma} - p_k\right] q_k\right] = \frac{1}{2}\left[\sum_{\forall k}\left[\frac{a}{b-3c} - p_k\right] q_k\right] \tag{6.2}$$

for $k = \{H_1CS, H_2CS\}$. This is indeed a special case of the previous expression, where equilibrium quantities of personal computer systems with open source software are zero.

Proposition 3. *Given market outcomes, consumer surplus under OS availability is equal to:*

$$CS^{os} = \frac{4a^2(b-c)^3(5b+c)}{(b-3c)\left[(b-3c)(7b-5c)+8c(b-c)\right]^2} \tag{6.3}$$

and when OS software is not developed:

$$CS^{nos} = \frac{a^2(b-2c)^2(b+c)}{(b-c)(b-3c)(3b-7c)^2} \tag{6.4}$$

Property 3. *Consumer surplus is greater under OS availability than when OS software is not developed,* i.e. $CS^{os} - CS^{nos} > 0$.

Proof. See Appendix F □

Consumer Surplus increases when OS is available, this is due to the greater variety of goods in the market. Although the price for composite goods which include CS software may increase[8], the effect of goods variety outweighed the increase in price in two of the four markets.

As it can be seen in Figure 3, the difference – which is always positive – in the two consumer surplus decreases with x (given, as before, $b = xc$ and $x > 3$). This means that the presence of OSS determines a relatively small increase in consumer surplus for relatively big values of x. This because, as seen in Property 2, the price of

[8]This is the case when $b > 14.8214c$ (Property 2).

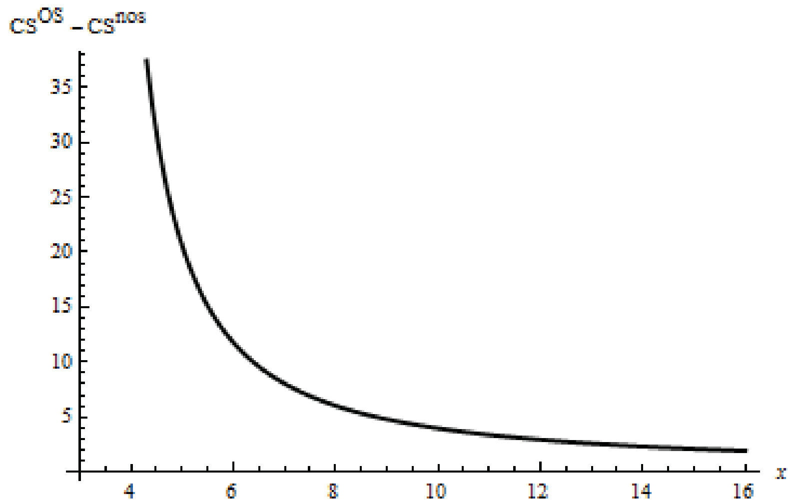

Figure 3: Difference of consumer surplus with regards of OSS availability as function of x, with $b = xc$, $x > 3$, $a = 5$ and $c = 0.35$

CSS–based bundles increases when OSS is available for big values of x ($x > 14.822$), leading to a smaller increase of consumer welfare under OSS availability.

7 First Best Benchmark

Before analysing welfare properties of equilibrium outcomes, it is useful to implement the first best optimum for the game.

This special case cannot be implemented, but its derivation is useful to analyse welfare properties of market equilibrium and second best solution.

Total welfare is considered as the unweighed sum of consumer surplus and industries' profits. In second stage, total welfare is maximised when prices are set equal to marginal costs, which are zero. The ex–ante expected total welfare (and consumer surplus) is maximised by social planner by setting the first best amount of contributions to open source.

Financing OS software is optimal, since consumer surplus under OS availability is bigger than that in the case OSS is not developed, as shown in Proposition 4.

Proposition 4. *Imposing all prices to be zero, Consumer Surplus is (with respect to the two R&D outcomes):*

$$CS^{os}_{FB} = \tfrac{2a^2}{b-3c} \quad ; \quad CS^{nos}_{FB} = \tfrac{a^2(b+c)}{(b-3c)(b-c)} \tag{7.1}$$

Moreover, CS under OS availability is greater than CS when OS software is not developed, i.e. $CS^{os}_{FB} > CS^{nos}_{FB}$.

Proof. See Appendix G □

Also in first best, under OSS availability consumer surplus (and total welfare) is greater than when OSS is not available. This is due to the presence of two more markets when OSS is released.

Total welfare corresponds to consumer surplus in this case. In first best, the *ex–ante* expected total welfare will be maximised[9].

$$F^{*}_{FB} = \arg\max_{F_{FB}} P(F_{TOT}) \cdot CS^{os}_{FB} + (1 - P(F_{TOT})) \cdot CS^{nos}_{FB} - F_{FB} \tag{7.2}$$

whose first order condition is:

$$[CS^{os}_{FB} - CS^{nos}_{FB}] \left. \frac{\partial P(F_{TOT})}{\partial F_{FB}} \right|_{F_{FB}=F^{*}_{FB}} = 1 \tag{7.3}$$

[9]Note that in this case $F_{TOT} = F_{FB} + \bar{F}$, since $\bar{F}$ represents exogenous contributions

Second order condition is satisfied because of concavity of the probability function. Furthermore, OS contribution is socially efficient if it improves the (expected) total welfare:

$$CS_{FB}^{nos} + P(F_{FB}^* + \bar{F})\,[CS_{FB}^{os} - CS_{FB}^{nos}] - F_{FB}^* > CS_{FB}^{nos} \tag{7.4}$$

Since consumer surplus under OS availability is bigger than CS_{FB}^{nos}, a positive contribution to OS is optimal (when condition (7.4) is satisfied).

The total amount of contribution is the sum of firms contributions, and considering the symmetric case[10], first order condition may be expressed as follows[11]:

$$[CS_{FB}^{os} - CS_{FB}^{nos}]\left.\frac{\partial P(F_{TOT})}{\partial F_{H_i}}\right|_{F_{H_i}=F_{FB}^*/2} = 1 \quad ; \quad i = 1, 2 \tag{7.5}$$

These conditions are indeed different from companies' first stage decisions (Expression n. 5.2).

Proposition 5. *The level of investment in first best is higher than in the equilibrium outcome.*

Proof. See Appendix H □

Since (hardware) firms consider only private benefits from OSS, and social benefits are greater than private ones, market equilibrium outcome leads to a smaller amount of investment. This comparison, however, is meaningless if in second stage is not possible to impose prices equal to marginal costs (zero). Moreover, in first best, corporate contributions are zero (because profits are zero). Consequently, when first

[10]By imposing $F_{FB} = F_{H_1} + F_{H_2}$ and $F_{H_1} = F_{H_2}$. Consequently $F_{FB} = 2F_{H_1} = 2F_{H_2}$.
[11]Since $\partial P(F_{TOT})/\partial F_{H_i} = \partial P(F_{TOT})/\partial F_{FB} \cdot \partial F_{FB}/\partial F_{H_i} = \partial P(F_{TOT})/\partial F_{FB}$

best cannot be reached – as it is assumed in this paper – second best optimum must be derived.

8 Second best optimum

It is assumed that an intervention on prices is not possible. In second best, total welfare is the sum of consumer surplus, hardware firms' profits and software producer's profits. The maximisation problem is[12]:

$$F_{SB}^{*} = \arg\max_{F_{SB}} P(F_{TOT}) \cdot TW^{os} \, (1 - P(F_{TOT})) \cdot TW^{nos} - F_{SB} \tag{8.1}$$

$$F_{SB}^{*} = \arg\max_{F_{SB}} TW^{nos} + P(F_{TOT}) \left[TW^{os} - TW^{nos} \right] - F_{SB} \tag{8.2}$$

By imposing $F_{SB} = F_{H_1} + F_{H_2}$, first order conditions are derived:

$$\left[TW^{os} - TW^{nos} \right] \left. \frac{\partial P(F_{TOT})}{\partial F_{H_i}} \right|_{F_{H_i} = F_{SB}^{*}/2} = 1 \quad ; \quad i = 1, 2 \tag{8.3}$$

Proposition 6. *The level of investment in second best is higher than in the equilibrium outcome,* i.e. $F_{SB}^{*} > 2F_{H_i}^{*}$.

Proof. See Appendix I □

The increase in hardware firms' profits due to the presence of open source software is less than the increase of total welfare, this creates a sub–optimal level of investments with respect to the second best optimum. Moreover, contributing to

[12]It is assumed that the total welfare is the sum of consumer surplus, hardware firm's profits and CS software company's profits, *i.e.* $TW^{os} = CS^{os} + 2\pi_{H_i}^{os} + \pi_{CS}^{os}$ and $TW^{nos} = CS^{nos} + 2\pi_{H_i}^{nos} + \pi_{CS}^{nos}$.

open source is profitable if and only if, total welfare increases, *i.e.*:

$$TW^{nos} - P(F_{SB}^{*} + \bar{F})\left[TW^{os} - TW^{nos}\right] - F_{SB}^{*} \geq TW^{nos} \tag{8.4}$$

In market equilibrium a positive private contribution may not exist even if it is socially optimal. This happens if the expected OS private gains (increase in hardware firms' profits) are smaller than investment in OSS development, while social gains – which are bigger than private ones – are greater than F_{SB}^{*}. This will be the case, if (See Appendix J):

$$\frac{P(F_{H_1}^{*} + F_{H_2}^{*} + \bar{F})}{F_{H_i^{*}}}\left[\pi_{H_i}^{os} - \pi_{H_i}^{nos}\right] < 1 \leq \frac{P(F_{SB}^{*} + \bar{F})}{F_{SB}}\left[TW^{os} - TW^{nos}\right] \tag{8.5}$$

Given the first order conditions in (5.2) and (8.3), condition (8.5) may be expressed in terms of elasticities as (See Appendix J):

$$\frac{\partial P(F_{SB}^{*} + \bar{F})}{\partial F_{H_i}}\frac{F_{SB}^{*}}{P(F_{SB}^{*} + \bar{F})} \leq 1 < \frac{F_{Hi}^{*}}{P(F_{H_1}^{*} + F_{H_2}^{*} + \bar{F})}\frac{\partial P(F_{H_1}^{*} + F_{H_2}^{*} + \bar{F})}{\partial F_{H_i}} \tag{8.6}$$

That is, the elasticity of the probability function in the market equilibrium is above one, while in second best optimum is smaller than one.

9 Public intervention

9.1 Availability of lump sum tax

Social planner may contribute to Open Source with a subsidy financed by a lump-sum tax. In this way, second best may be reached by a contribution F_{lstax} (and an equal lump sum tax) such that:

$$F^*_{lstax} = F^*_{SB} - 2F^*_{H_i} \tag{9.1}$$

This contribution permits to reach the second best optimum. The expected total welfare is maximised when:

$$F^*_{lstax} = \arg\max_{F_{lstax}} TW^{nos} - P(2F^*_{H_i} + F_{lstax} + \bar{F})\left[TW^{os} - TW^{nos}\right] - 2F^*_{H_i} - F_{lstax} \tag{9.2}$$

Since F^*_{SB} is the amount of total contribution which maximises total welfare, imposing $F^*_{lstax} = F^*_{SB} - 2F^*_{H_i}$ leads to the second best optimum. It is assumed that this intervention is financed by a tax which does not create excess of burden of taxation, but note that F^*_{lstax} represents the total amount of revenues raised through the lump sum tax. We do not specify how lump sum tax is divided among agents.

9.2 Contribution financed through corporate income tax

In this section, public contribution to open source is financed by corporate taxes on both hardware and software firms. The model is modified adding a new stage – namely stage zero – in which the social planner sets the tax rate t (with $t \in (0, 1)$)

on profits and the amount of public contribution F_{ctax}. Then, first and second stage take place in the same fashion of previous pages.

Price competition (second stage decision) is not affected by the corporate tax, since it does not affect equilibrium prices. Equilibrium profits, however, are a fraction $1-t$ of previous profits.

First stage decision – which refers to private (hardware firms') donations to open source – is affected by the public intervention, as we will see later. A generic hardware firm will maximise:

$$F_{H_i}^{ctax} = \arg\max_{F_{Hi}} \quad \pi_{Hi}^{nos}(1-t)$$
$$+P(F_{H_i}+F_{H_j}+F_{ctax}+\bar{F})\left[\pi_{Hi}^{os}-\pi_{Hi}^{nos}\right](1-t)-F_{Hi}(1-t) \tag{9.3}$$

for $i,j=1,2$ and $i\neq j$. First order condition is:

$$\left[\pi_{Hi}^{os}-\pi_{Hi}^{nos}\right](1-t)\left.\frac{\partial P(F_{TOT})}{\partial F_{Hi}}\right|_{F_{Hi}=F_{Hi}^{ctax}}=1-t \tag{9.4}$$

Finally, social planner maximises the social welfare by setting an optimal tax rate t, and giving a contribution F_{ctax}. Contributions must satisfy the public budget constraint in expression (9.5).

$$F_{ctax}=\left\{\pi_{CS}^{nos}+2\pi_{Hi}^{nos}+P(2F_{H_i}^{ctax}+F_{ctax}+\bar{F})\left[\pi_{CS}^{os}+2\pi_{Hi}^{os}-\pi_{CS}^{nos}-2\pi_{Hi}^{nos}\right]-2F_{H_i}^{ctax}\right\}t \tag{9.5}$$

Therefore the tax rate may be expressed in the following fashion:

$$t = \frac{F_{ctax}}{\pi^{nos}_{CS} + 2\pi^{nos}_{Hi} + P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\left[\pi^{os}_{CS} + 2\pi^{os}_{Hi} - \pi^{nos}_{CS} - 2\pi^{nos}_{Hi}\right] - 2F^{ctax}_{H_i}} \tag{9.6}$$

Finally, the social maximisation problem is:

$$\begin{aligned} F^*_{ctax} = \arg\max_{F_{ctax}} \quad & \left[CS^{nos} + (\pi^{nos}_{CS} + 2\pi^{nos}_{Hi})(1-t)\right] \\ & +P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\left[CS^{os} + (\pi^{os}_{CS} + 2\pi^{os}_{Hi})(1-t)\right. \\ & \left.-CS^{nos} - (\pi^{nos}_{CS} + 2\pi^{nos}_{Hi})(1-t)\right] - 2F^{ctax}_{H_i}(1-t) - F_{ctax} + F_{ctax} \end{aligned} \tag{9.7}$$

It easy to show that the above maximisation problem is equivalent to the following problem (see Appendix K):

$$\begin{aligned} F^*_{ctax} = \arg\max_{F_{ctax}} \quad & TW^{nos} + P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\left[TW^{os} - TW^{nos}\right] \\ & -2F^{ctax}_{H_i} - F_{ctax} \end{aligned} \tag{9.8}$$

As before, imposing $2F^{ctax}_{H_i} + F_{ctax} = F^*_{SB}$ leads to the second best outcome, therefore the optimal public contribution will be:

$$F^*_{ctax} = F^*_{SB} - 2F^{ctax}_{H_i} \tag{9.9}$$

and the optimal tax rate will be calculated accordingly using expression (9.6).

Property 4. *Under corporate income taxation, the amount of private contributions is smaller than the equilibrium outcome without public intervention,* i.e. $F^{*}_{Hi} > F^{ctax}_{Hi}$.

Proof. See Appendix L. □

Income tax and public transfers to the OS Foundation create a distortion in the amount of private contributions. Public intervention has a crowding out effect on private investments in OSS. Despite this distortion in private contributions, second best optimum can be reached also without a lump sum tax by imposing a certain tax rate t on firms' profits. However, the optimal corporate income tax rate t^{*} should be less than – or equal to – 1 ($t^{*} \leq 1$). From expression (9.6), this means that:

$$\pi^{nos}_{CS} + 2\pi^{nos}_{Hi} + P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\left[\pi^{os}_{CS} + 2\pi^{os}_{Hi} - \pi^{nos}_{CS} - 2\pi^{nos}_{Hi}\right] - 2F^{ctax}_{H_i} \geq F^{*}_{ctax} \tag{9.10}$$

$$\pi^{nos}_{CS} + 2\pi^{nos}_{Hi} + P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\left[\pi^{os}_{CS} + 2\pi^{os}_{Hi} - \pi^{nos}_{CS} - 2\pi^{nos}_{Hi}\right] \geq F^{*}_{SB} \tag{9.11}$$

Expected profits of the two industries (the left hand side of expression (9.11)) must be greater than the optimal total second best contribution. Indeed this is needed because public contributions to the OSS foundation will be raised using industries' profits .

So, although corporate income tax may be optimal (in second best), there could be cases where second best optimum cannot be reached using this instrument.

10 Conclusions

This paper contributes to the growing literature regarding open source phenomenon. In this model, the complementarity between hardware and software provides an incentive for corporate contributions to OSS development.

Results are several. The release of OSS has a positive impact on hardware firms' profits and prices, as well as on consumer surplus. Furthermore, it has a negative effect on CS software firm's profits and prices. The total effect on social welfare is positive.

OSS release has two major positive consequences in our model. On the one hand, the absence of OSS determine a lower welfare, since there are consumers who may prefer OSS bundles rather than CSS-based PCS. Only some of them substitute their consumption of personal computer systems with CSS-based bundles. On the other hand, since open source is free, the problem of double mark-up is partially solved (in two of the four markets). Furthermore, price competition for software is more intense, due the public availability of OSS. OSS development has always a positive impact on total welfare.

Hardware firms – under the non negativity condition – contribute positively to the development of open source software. This contribution, however, is not optimal with respect to the second best benchmark. Second best can be reached with a lump sum tax, and with a corporate income tax. With the latter instrument, however, second best may not be a feasible point if the two industries' profits are less than the optimal second best total contribution.

This model may explain the recent development of several open source operating systems for embedded devices (such as tablets, smartphones, netbooks, etc.), since hardware firms incentives increase when the market is relatively homogeneous, and competition is more intense.

A limit of the model is that we do not consider the possibility that hardware firms can decide to develop their own closed source operating system. With OSS, however, R&D development has greater chances of being successful, since donations by firms (as well as exogenous and public contributions) jointly increase the probability of software development.

Other critical points are the assumption of compatibility of hardware and software, as well as the assumed symmetric demand for CSS and OSS bundles. Future developments may consider the interaction between a greater number of software companies, as well as differences in the quality of software.

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Appendix

Note that Proof using *Mathematica* are available in a separate document, which can be released on request by the author.

A Proof of Property 1

It can be easily shown that the following inequalities hold:

$\alpha = a\frac{b+c}{b-c} > a$, since $\frac{b+c}{b-c} > 1$;

$\beta = \frac{(b-2c)(b+c)}{b-c} < b$, since $(b-2c)(b+c) < b(b-c) \iff 2c^2 > 0$;

$\gamma = c\frac{b+c}{b-c} > c$, since $\frac{b+c}{b-c} > 1$;

$\beta - \gamma = \frac{(b-3c)(b+c)}{b-c} < b - c$, since $(b-3c)(b+c) < (b-c)^2 \iff 4c^2 > 0$.

B Equilibrium when OSS is not released

Equilibrium, when OSS is not available, (expression n. 4.8) may also be expressed using original parameters a, b and c:

$$p_{CS}^{nos} = \frac{a(b-2c)}{(b-3c)(3b-7c)}; \qquad q_{CS}^{nos} = 2\frac{a(b+c)(b-2c)}{(b-c)(3b-7c)}; \qquad \pi_{CS}^{nos} = 2\frac{a^2(b+c)(b-2c)^2}{(b-c)(3b-7c)^2(b-3c)}$$
$$p_{H_1}^{nos} = p_{H_2}^{nos} = \frac{a}{(3b-7c)}; \quad q_{H_1}^{nos} = q_{H_2}^{nos} = \frac{a(b+c)(b-2c)}{(b-c)(3b-7c)}; \quad \pi_{H_1}^{nos} = \pi_{H_2}^{nos} = \frac{a^2(b+c)(b-2c)}{(b-c)(3b-7c)^2} \tag{B.1}$$

Furthermore, due to the market structure, the following equalities hold true: $q_{H_1CS}^{nos} = q_{H_2CS}^{nos} = q_{H_1}^{nos} = q_{H_2}^{nos} = \frac{1}{2}q_{CS}^{nos}$ and:

$$p_{H_1CS}^{nos} = p_{H_2CS}^{nos} = p_{H_1}^{nos} + p_{CS}^{nos} = p_{H_2}^{nos} + p_{CS}^{nos} = \frac{a(2b-5c)}{(b-3c)(3b-7c)} \tag{B.2}$$

C Equilibrium quantities for composite goods under OS availability

Given maximisation problems in expressions (4.9) and (4.11), the system of first order conditions leads to the following equilibrium quantities – and prices – for the personal computer systems (composite goods):

$$
\begin{aligned}
q^{os}_{H1CS} = q^{os}_{H2CS} &= \tfrac{2a(b-c)^2}{(b-3c)(7b-5c)+8c(b-c)} \\
q^{os}_{H1OS} = q^{os}_{H2OS} &= \tfrac{4ab(b-c)}{(b-3c)(7b-5c)+8c(b-c)} \\
p^{os}_{H1CS} = p^{os}_{H2CS} &= \tfrac{a(5b-3c)}{(b-3c)(7b-5c)+8c(b-c)} \\
p^{os}_{H1OS} = p^{os}_{H2OS} &= \tfrac{a(3b-c)}{(b-3c)(7b-5c)+8c(b-c)}
\end{aligned}
\tag{C.1}
$$

Expressing quantities and prices in this fashion is useful to compute consumer surplus.

D Proof of Property 2

Due to symmetry of demand functions, prices and quantities for the two hardware companies are equal. The first inequality is always satisfied[13]:

$$\begin{aligned} p_{Hi}^{os} - p_{Hi}^{nos} &> 0 \\ &= \frac{a(3b-c)}{(b-3c)(7b-5c)+8c(b-c)} - \frac{a}{3b-7c} \\ &= a\frac{(3b-c)(3b-7c)-(b-3c)(7b-5c)-8c(b-c)}{(3b-7c)((b-3c)(7b-5c)+8c(b-c))} \\ &= a\frac{2b(b-3c)}{(3b-7c)((b-3c)(7b-5c)+8c(b-c))} > 0 \end{aligned} \tag{D.1}$$

CS software price decreases when OS software is available, *i.e.*:

$$\begin{aligned} p_{CS}^{nos} - p_{CS}^{os} &> 0 \\ &= \frac{a(b-2c)}{(3b-7c)(b-3c)} - \frac{2a(b-c)}{(b-3c)(7b-5c)+8c(b-c)} \\ &= a\frac{(b-2c)(b-3c)(7b-5c)+8c(b-2c)(b-c)-2(b-c)(3b-7c)(b-3c)}{(3b-7c)(b-3c)[(b-3c)(7b-5c)+8c(b-c)]} \\ &= a\frac{(b-3c)(b+c)(b-c)+c\left[(b-3c)^2+8(b-2c)(b-c)\right]}{(3b-7c)(b-3c)[(b-3c)(7b-5c)+8c(b-c)]} > 0 \end{aligned} \tag{D.2}$$

Hardware firms' profits (without considering R&D costs) increase when OS is available, and CS software producer's profits decrease when OS software is available,

[13]Recall that composite goods are gross substitutes and, therefore, $b > 3c$

therefore:

$$\begin{aligned}\pi_{Hi}^{os} - \pi_{Hi}^{nos} &= \tfrac{2a^2(b-c)(3b-c)^2}{[(b-3c)(7b-5c)+8c(b-c)]^2} - \tfrac{a^2(b+c)(b-2c)}{(3b-7c)^2(b-c)} > 0 \\ \pi_{CS}^{nos} - \pi_{CS}^{os} &= 2\tfrac{a^2(b+c)(b-2c)^2}{(b-c)(3b-7c)^2(b-3c)} - \tfrac{8a^2(b-c)^3}{((b-3c)(7b-5c)+8c(b-c))^2} > 0\end{aligned} \tag{D.3}$$

Due to complexity of the polynomials, the computational software *Mathematica* is used to verify these two conditions; which are always true for $b > 3c$, $a > 0$ and $c > 0$. Finally, the difference in the price of PC systems with CS software, with or without OS software, is:

$$p_{HiCS}^{nos} - p_{HiCS}^{os} = \tfrac{a(2b-5c)}{(b-3c)(3b-7c)} - \tfrac{a(5b-3c)}{(b-3c)(7b-5c)+8c(b-c)} \tag{D.4}$$

Using *Mathematica*, p_{HiCS}^{nos} is greater than p_{HiCS}^{os} if $b^3 + 49bc^2 < 18b^2c + 28c^3$ or, equivalently,:

$$p_{HiCS}^{nos} > p_{HiCS}^{os} \iff b^2(b - 18c) + c^2(49b - 28c) < 0 \tag{D.5}$$

Imposing $b = xc$ with $x > 3$, the inequality in expression (D.5) can be rewritten as $c^3\left[x^2(x - 18) + 49x - 28\right] < 0$. The parameter c is greater than zero by assumption. The polynomial in the square brackets is equal to zero ($x^2(x - 18) + 49x - 28 = 0$)

when x assumes the following values:

$$\{\{x \to 0.791373\}, \{x \to 2.38719\}, \{x \to 14.8214\}\} \tag{D.6}$$

In the interval of our interest, *i.e.* for $x > 3$, the polynomial is smaller than zero ($x^2(x-18)+49x-28 < 0$) for $x < 14.8214$. While is greater than zero when $x > 14.8214$. This can be seen in Figure 4.

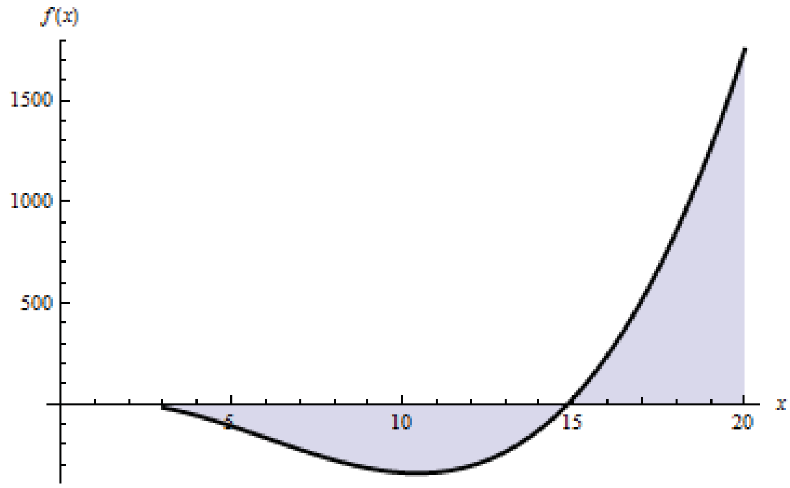

Figure 4: Plot of $f(x) = x^2(x-18)+49x-28$ for values $x > 3$

When b is "big enough" ($b > 14.8214c$) then $p^{nos}_{HiCS} < p^{os}_{HiCS}$. Therefore, when the own price derivative is relatively big, the effect on hardware price (which increases under OS availability) outweighs the decrease of CSS price. The overall effect is an increase in the composite good's price.

E Consumer surplus

The system of equation in expression (3.1) shows demands as a function of prices. In the following expression (E.1), the system is rewritten in order to have prices as a function of quantities:

$$p_{H_1CS} = \frac{a}{b-3c} - \frac{b-2c}{(b-3c)(b+c)} q_{H_1CS} - \frac{c}{(b-3c)(b+c)} \left[q_{H_2CS} + q_{H_1OS} + q_{H_2OS}\right]$$

$$p_{H_2CS} = \frac{a}{b-3c} - \frac{b-2c}{(b-3c)(b+c)} q_{H_2CS} - \frac{c}{(b-3c)(b+c)} \left[q_{H_1CS} + q_{H_1OS} + q_{H_2OS}\right]$$

$$p_{H_1OS} = \frac{a}{b-3c} - \frac{b-2c}{(b-3c)(b+c)} q_{H_1OS} - \frac{c}{(b-3c)(b+c)} \left[q_{H_2CS} + q_{H_1CS} + q_{H_2OS}\right] \tag{E.1}$$

$$p_{H_1OS} = \frac{a}{b-3c} - \frac{b-2c}{(b-3c)(b+c)} q_{H_2OS} - \frac{c}{(b-3c)(b+c)} \left[q_{H_2CS} + q_{H_1OS} + q_{H_1CS}\right]$$

Then consumer surplus in expression (6.1) is easily derived using expression (4) in Cellini et al. (2004).

F Proof of Property 3

Using *Mathematica* it is easy to verify that the following expression holds true for $a > 0$, $b > 3c$ and $c > 0$:

$$CS^{os} - CS^{nos} = \frac{4a^2(b-c)^3(5b+c)}{(b-3c)[(b-3c)(7b-5c)+8c(b-c)]^2} - \frac{a^2(b-2c)^2(b+c)}{(b-c)(b-3c)(3b-7c)^2} > 0 \tag{F.1}$$

G Proof of Proposition 4

When OS software is not available, and prices are zero, quantities demanded are $q_{H1CS} = q_{H2CS} = \alpha$. Therefore the consumer surplus is:

$$CS_{FB}^{nos} = \frac{1}{2} \cdot 2 \left[\frac{\alpha}{\beta - \gamma} - 0 \right] \alpha = \frac{\alpha^2}{\beta - \gamma} = \frac{a^2(b+c)}{(b-c)(b-3c)} \tag{G.1}$$

Industries' profits are zero. Equivalently, under OS availability, $q_{H1CS} = q_{H2CS} = q_{H1OS} = q_{H2OS} = a$. Consumer surplus is:

$$CS_{FB}^{os} = \frac{1}{2} \cdot 4 \left[\frac{a}{b-3c} - 0 \right] a = \frac{2a^2}{(b-3c)} \tag{G.2}$$

The difference between the two surpluses is:

$$CS_{FB}^{os} - CS_{FB}^{nos} = \frac{2a^2}{(b-3c)} - \frac{a^2(b+c)}{(b-c)(b-3c)} = \frac{a^2}{b-c} \tag{G.3}$$

which is greater than zero.

H Proof of proposition 5

First order conditions, respectively, in market equilibrium and in first best are shown in expression (5.2) and (7.5). Since the second derivative of the probability function is negative, there is a smaller amount of investments with respect to the first best if:

$$\left.\frac{\partial P(F_{TOT})}{\partial F_{H_i}}\right|_{market} > \left.\frac{\partial P(F_{TOT})}{\partial F_{H_i}}\right|_{FB} \quad ; i = 1, 2 \tag{H.1}$$

Which is equivalently to say that the first derivative of $P(\cdot)$ in the market outcome is greater than that in First best (with prices equal to zero). This condition can be rewritten as follows (at optimum):

$$\frac{1}{\pi_i^{os}-\pi_i^{nos}} > \frac{1}{CS_{FB}^{os}-CS_{FB}^{nos}} \qquad ; i = 1,2$$

$$CS_{FB}^{os} - CS_{FB}^{nos} - [\pi_i^{os} - \pi_i^{nos}] > 0 \quad ; i = 1,2 \tag{H.2}$$

Using the computational software, it can be easily seen that this condition is satisfied for $a > 0$, $b > 3c > 0$.

I Proof of proposition 6

In the same fashion of the previous proof, it is necessary to demonstrate that:

$$\left.\frac{\partial P(F_{TOT})}{\partial F_{H_i}}\right|_{market} > \left.\frac{\partial P(F_{TOT})}{\partial F_{H_i}}\right|_{SB} \quad ; i = 1,2 \tag{I.1}$$

Which corresponds to the following inequality (at optimum):

$$\frac{1}{\pi_i^{os}-\pi_i^{nos}} > \frac{1}{TW^{os}-TW^{nos}} \qquad ; i = 1,2$$

$$\left[CS^{os} + 2\pi_{H_i}^{os} + \pi_{CS}^{os} - \left(CS^{nos} + 2\pi_{H_i}^{nos} + \pi_{CS}^{nos}\right)\right] - [\pi_i^{os} - \pi_i^{nos}] > 0 \quad ; i = 1,2 \tag{I.2}$$

Which holds true in the parameters' domain $a > 0$, $b > 3c > 0$.

J Provision of OS

Contributions will not be devolved to the OS foundation if (from expression n. 5.3):

$$P(F^*_{H_1} + F^*_{H_2} + \bar{F})\frac{[\pi^{os}_{Hi} - \pi^{nos}_{Hi}]}{F^*_{Hi}} < 1 \tag{J.1}$$

While a positive contribution is socially optimal if (expression n. 8.4):

$$P(F^*_{SB} + \bar{F})\frac{[TW^{os} - TW^{nos}]}{F^*_{SB}} \geq 1 \tag{J.2}$$

Note, furthermore, that – according to Proposition n. 6 – the inequality $F^*_{SB} > 2F^*_{H_i}$, where F^*_{SB} is the SB optimal amount of total contribution and $2F^*_{H_i}$ is the total amount of contribution in market equilibrium.

OS software will not be financed although it is socially optimal (in second best) to do so, if:

$$P(F^*_{H_1} + F^*_{H_2} + \bar{F})\frac{[\pi^{os}_{Hi} - \pi^{nos}_{Hi}]}{F^*_{Hi}} < 1 \leq P(F^*_{SB} + \bar{F})\frac{[TW^{os} - TW^{nos}]}{F^*_{SB}} \tag{J.3}$$

By substituting the first order conditions, respectively, of individuals firms and of social planner (expressions n. 5.2 and 8.3), the above expression could be rewritten as:

$$P(F^*_{H_1} + F^*_{H_2} + \bar{F}) \frac{1}{F^*_{Hi} \frac{\partial P(F^*_{H_1} + F^*_{H_2} + \bar{F})}{\partial F_{H_i}}} < 1 \leq P(F^*_{SB} + \bar{F}) \frac{1}{F^*_{SB} \frac{\partial P(F^*_{SB} + \bar{F})}{\partial F_{H_i}}} \tag{J.4}$$

Which can be rewritten in form of elasticities:

$$\frac{\partial P(F^*_{SB} + \bar{F})}{\partial F_{H_i}} \frac{F^*_{SB}}{P(F^*_{SB} + \bar{F})} \leq 1 < \frac{F^*_{Hi}}{P(F^*_{H_1} + F^*_{H_2} + \bar{F})} \frac{\partial P(F^*_{H_1} + F^*_{H_2} + \bar{F})}{\partial F_{H_i}} \tag{J.5}$$

K Welfare maximisation under corporate income tax

Equation (9.7) can be expressed as:

$$\begin{aligned} F^*_{ctax} = \arg\max_{F_{ctax}} \quad & CS^{nos} - P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F}) \left[CS^{os} - CS^{nos} \right] \\ & + \{ \pi^{nos}_{CS} + 2\pi^{nos}_{Hi} \\ & + P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F}) \left[\pi^{os}_{CS} + 2\pi^{os}_{Hi} - \pi^{nos}_{CS} - 2\pi^{nos}_{Hi} \right] - 2F^{ctax}_{H_i} \Big\} (1 - t) \end{aligned} \tag{K.1}$$

Given budget constraints in expressions (9.5) and (9.6), the above expression can be rewritten as:

$$F^*_{ctax} = \arg\max_{F_{ctax}} \quad CS^{nos} - P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\,[CS^{os} - CS^{nos}]$$
$$+\pi^{nos}_{CS} + 2\pi^{nos}_{Hi}$$
$$+P(2F^{ctax}_{H_i} + F_{ctax} + \bar{F})\,[\pi^{os}_{CS} + 2\pi^{os}_{Hi} - \pi^{nos}_{CS} - 2\pi^{nos}_{Hi}] - F_{ctax} - 2F^{ctax}_{H_i} \tag{K.2}$$

Which can be rewritten as in expression (9.8), because $TW^{nos} = CS^{nos} + \pi^{nos}_{CS} + 2\pi^{nos}_{Hi}$ and $TW^{os} = CS^{os} + \pi^{os}_{CS} + 2\pi^{os}_{Hi}$.

L Proof of Property 4

Using the first order condition in expressions (5.2) (on the left hand side) and (9.4) (on the right hand side), we have:

$$\left.\frac{\partial P(2F_{H_i} + \bar{F})}{\partial F_{H_i}}\right|_{F_{H_i} = F^*_{H_i}} = \frac{1}{\pi^{os}_{H_i} - \pi^{nos}_{H_i}} = \left.\frac{\partial P(2F_{H_i} + F^*_{ctax} + \bar{F})}{\partial F_{H_i}}\right|_{F_{H_i} = F^{ctax}_{H_i}} \tag{L.1}$$

Since: (1) the two derivatives, in the evaluation points, must be equal; (2) $F^*_{SB} = 2F^*_{H_i} + F^*_{ctax} > 2F^*_{H_i}$, and (3) the second derivative of the probability function is negative; the equality holds if and only if $F^*_{Hi} > F^{ctax}_{Hi}$.

Chapter 2

Strategic patent complexity: entry deterrence and other anti–competitive behaviours

Abstract

Several authors have shown how patent breadth decision can be used to deter entry. In a context where the patent breadth is not the only choice for the firm, we want to analyse if patent complexity – which has the effect of hiding information about the actual patent breadth – can have anti–competitive effects.

In this paper we examine incumbent's strategic choice of patent breadth and complexity in a horizontal differentiated market. Patent complexity determines the degree of information spillovers, released to the potential entrant. Patent breadth and complexity, together, increase incumbent's probability of loosing an infringement/patent validation challenge. Results are several.

Entry deterrence is possible for certain values of parameters. There is an incentive in increasing the complexity, although, there exists an upper–bound which is inefficient to cross, it represents a credibility bound. Finally, an infringement trial never occurs.

JEL Classification: L11; D21; D82

1 Introduction

A huge interest in economics literature has been put on innovation and patent. The set up of a patent system has the primary aim of incentive innovation among firms by giving market power to innovators. Therefore, there exists a trade–off between benefits deriving from innovation, and welfare losses due to market power.

Firms can strategically behave – using patent – to lessening competition in a certain market. Gilbert and Newbery [1982], for instance, analyse preemptive patents which are used, by a dominant firm, to gain monopolistic power.

In the literature we find two main dimensions to define a patent: patent length and breadth [Merges and Nelson, 2007, Klemperer, 1990, Gilbert and Shapiro, 1990]. While patent length refers to the amount of time the patent protection is in force, there are several definitions of patent breadth. Gilbert and Shapiro [1990], for instance, define patent breadth as *"the flow rate of profit available to the patentee while the patent is in force"*, linking patent breadth definition with market power exerted by incumbent. Robledo [2005], on the other hand, defines patent width as the *"scope of the patent"*. Yiannaka and Fulton [2006, 2011] model a vertically differentiated market, where the patent breadth is identified as a unidimensional characteristic (quality) dimension. In their model, patent guarantees monopoly power over a certain proportion of this unidimensional characteristic.

Asymmetric information may affect competition as well. Private information can be exploited by an incumbent firm, and results in strategic anti–competitive behaviours, such as entry deterrence. Milgrom and Roberts [1982], for instance, explore entry deterrence due to asymmetric information when the potential entrant's knowledge about pay-off is limited, and price could be a signal of this information. Similarly, Matthews and Mirman [1983] build a model in which the price is a signal of industry characteristics and stochastic shocks influence incumbent's price. They find that equilibrium results in limiting pricing, lower entry probability, as well as lower entrant's profits. Harrington [1987] analyses the case of asymmetric industry information, when the initial industry structure is an oligopoly. He finds that the bigger is the number of potential entrants, the greater is the pre–entry price and the lower is the entry likelihood.

In this paper we seek to jointly consider patent and asymmetric information. Our aim is to analyse incumbent's strategic behaviour, when it is possible to patent a product and hide some information about the patent breadth. In particular, we introduce the concept of patent complexity.

Although Crouch [2008] briefly analyse the increasing size and complexity of patents – through a content analysis aiming at counting the number of words in the description of the patent excluding *"claims, title, abstract, references, and other identifying information"* [Crouch, 2008] – there is no definition in literature for patent complexity.

The concept of patent complexity is linked to both product structure and information revealed by the patent. Technological improvements lead to more complicated products. Therefore, complexity is a consequence of evolution of products. However, a great deal of complexity could have a negative effect on information released to external agents. Competitors use information from the patent to define the boundaries of protection, so they can set their competitive strategies accordingly. Complexity, however, may have an effect on signals received by competitors. Perhaps, the more complex is the patent, the more uncertain are those boundaries; and uncertainty could result in less competition.

In our opinion, for a certain type of products/inventions, patent complexity may affect the perceived breadth of protection granted, and the level of uncertainty about the patent itself. This model could be useful to describe the anti–competitive behaviours when the complexity of the patent does not permit to understand the actual patent breadth, therefore potential entrants prefer not to enter or sustain a greater amount of costs because of the threat of the trial.

Yiannaka and Fulton [2006] argue that patent breadth choice has a trade–off, given by the market power resulting from patent protection, and the probability of infringements and validity challenges, which have an uncertain outcome. They argue that, since the efficient patent system hypothesis is unlikely to hold, the wider is the breadth protection, the smaller are the chances of winning the trial for the incumbent. Barton [2000] analyses the effectiveness in evaluating and assessing patent claims in the United States Patent and Trademark Office, which decreases because of time constraints. If the patent system is not efficient, patent could be granted even if they *"cannot survive a validity attack, thus leading to disputes that have to be resolved through costly litigation or settlement"* [Yiannaka and Fulton, 2006].

We develop a model where an incumbent decide to patent a certain product. He chooses patent breadth and complexity. The breadth, which in in Yiannaka and Fulton [2006, 2011] is a quality dimension, refers to a vertical differentiation characteristic protected by the patent. It could be, for example, the design or the specifics of the product. We add complexity, which has an effect on information signals transmitted to potential entrant.

Our definition of patent complexity refers only to the way in which information is stated in the patent. Complexity may lead to the concealment of information, to uncertain boundaries of the patent breadth, and to a bigger likelihood of infringement threats by the incumbent.

This paper analyses patent breadth and complexity decision by an incumbent with drastic innovation. This decision reflects a strategic behaviour of the incumbent/monopolist. As in Yiannaka and Fulton [2006, 2011], we use a context of drastic innovation, because of the relevance of R&D costs, of industry profits and, finally, for the likelihood that the patent office is less reliable in assessing the patent claim (thus weakening the efficient patent system hypothesis). We assume that the patent office plays no role in assessing the patent claim and determining the breadth. Patent office accepts always the claim and grants the patent. We stress that complexity, in our model, affects only information signals about the breadth. Due to inefficiencies of the patent system, an infringement or validity challenge could have an uncertain outcome. The wider is the breadth granted and the higher is the degree of complexity, the lower are incumbent's probabilities of winning the trial (if it takes place). Thus, the chances that a patent is declared invalid increase with complexity and breadth.

The basic setting is derived from Yiannaka and Fulton [2006, 2011]. Differently from Yiannaka and Fulton [2006], we do not specify a function for the probability of winning an infringement/validation trial and use a generic function[1]. The infringement/trial occurs always when there is infringement, as in Yiannaka and Fulton [2006].

Our results are several. Patent complexity has two major effects. On the one hand, it increases the effect of the breadth because potential entrant is uncertain about the real protection granted. On the other hand, there are problems on the credibility of the patent threat. So, there exists an upperbound, above which potential entrant considers patent breadth non credible.

The paper is organised as follow. In the first part we describe the game. Then we will analyse the model and derive the Perfect Bayesian Equilibrium for the game. The paper concludes with some final comments and remarks about the model.

2 The model

The model of entry deterrence with patent complexity is a sequential game with incomplete information. There is an incumbent and a potential entrant (indexed by I and E) who compete in a vertically differentiated market. Both firms are risk neutral.

The incumbent decides whether to patent its invention or not. The probability of granting a claimed patent with a patent breadth $b > 0$ is always one.

We define complexity as a positive parameter ν. Complexity, as we model it, creates a disturbance on the mechanism of information spillovers from the incumbent

[1] While in Yiannaka and Fulton [2006] this probability is specified and there is a closed form solution, in their second paper [Yiannaka and Fulton, 2011] the same authors use a generic function, leading to more general results.

to the entrant.

Definition 1. *Define a continuous function* $\hat{b} = h(\nu)$*. It is the signal, which the entrant receives, about the maximum value which can be assumed by the patent breadth. This function is increasing, i.e.* $\frac{\partial \hat{b}}{\partial \nu} > 0$*. Moreover, we have that* $\lim_{\nu \to 0} \hat{b} = 0$ *and* $\hat{b} \leq 1$*.*

We assume complexity to have a strong effect on information spillovers. Potential entrant does not know the real patent breadth (which is below $\hat{b}$), he receives the signal that the patent breadth is located in the interval between zero (not included) and $\hat{b}$.

We are assuming that potential entrant is not able to determine how wide is the actual protection granted through the patent. Complexity, moreover, decreases the amount of information revealed to the entrant, affecting entrant's evaluation process. The greater is complexity, the wider is the interval in which the actual breadth is located.

We can immagine that, given a certain breadth b, the greater is the complexity the more difficult is for an external agent to understand the real breadth. In our fashion, we hike this effect by assuming that the only information transmitted is the upper bound $\hat{b}$. Therefore, the entrant receives no information about the actual patent breadth but only a signal regarding its maximum possible value. The greater is ν, the greater is $\hat{b}$ and the wider is the range within which the real patent breadth is located.

In the market there is a mass of one of consumers, each of them has utility:

$$U_i = U + \lambda x_i - p_i; \quad i = \{I, E\} \tag{2.1}$$

Where λ is a taste parameter, distributed in the population according to a certain distribution $F(\lambda)$. For seek of simplicity, we assume that λ follows a uniform distribution between 0 and 1 ($\lambda \sim U[0,1]$). Parameters x_i and p_i are, respectively, the quality and the price of firm i's product.

The parameter x_I is normalised to 0. Consequently, x_E is interpreted as the relative differentiation degree with respect to x_I. Quality parameter is bounded between 0 and 1. Given p_I, x_E and p_E, consumers will buy from I if $U_I \geq U_E$. Thus, only if:

$$U - p_I \geq U + \lambda x_E - p_E \iff \lambda \leq \frac{p_E - p_I}{x_E}$$

Demand of incumbent's products is equal to the probability that λ is smaller than $[p_E - p_I]/x_E$[2]. Therefore, demand functions, of I and E, are:

$$q_I = Pr\left(\lambda \leq \tfrac{p_E - p_I}{x_E}\right) = \tfrac{p_E - p_I}{x_E}; \quad q_E = Pr\left(\lambda \geq \tfrac{p_E - p_I}{x_E}\right) = 1 - q_I = \tfrac{x_E - p_E + p_I}{x_E} \tag{2.2}$$

Incumbent has already sustained R&D costs, which are sunk. It could decide to patent the innovation or not, paying a certain fee τ.

As in Yiannaka and Fulton [2006, 2011], a certain patent, with breadth b, assures to the incumbent protection against competitors, for all products with a differentia-

[2]Since the mass of consumers is one. Note also that λ is distributed as a uniform random variable in $[0,1]$.

tion parameter below b. If, however, the entrant sets $x_E < b$, an infringement challenge will take place. A patent is always granted, regardless to its breadth and complexity. Last assumption derives from non–efficiency of the patent system, therefore a patent granted may be considered invalid *ex–post*, in the infringement/validation trial.

Since we want to focus our attention on incumbent's anti–competitive behaviour, we assume that there is only a potential entrant. So, the entrant does not have to decide whether patent its innovation or not. The marginal cost to produce the good is equal for both firms and is normalised to zero. Entrant sustains R&D costs $C(x_E) = \frac{\beta}{2}x_E^2$ to develop the new product with a certain quality x_E.

The game is divided in three stages (Figure n. 1).

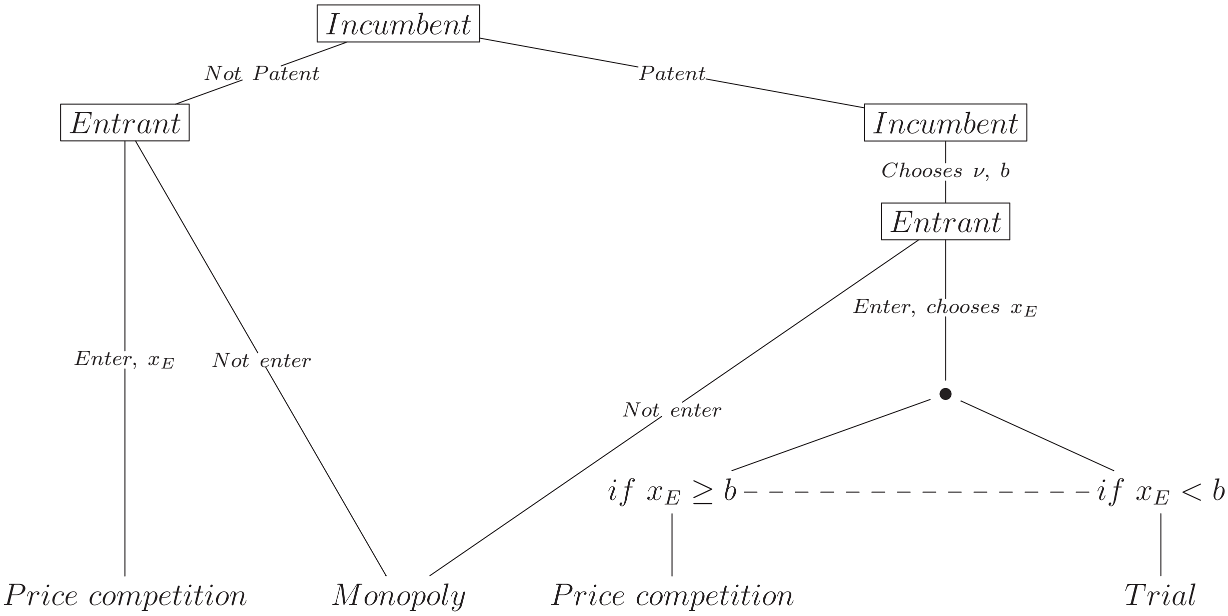

Figure 1: Representation of the Patenting Game

In the first stage, the product development process already took place, and the incumbent decides to patent the invention or not (if he patents, the product he will choose the patent breadth and complexity). There are administrative costs ($\tau > 0$) to be sustained to patent the invention. After patenting the product, the actual breath b will be not known by the entrant. He, instead, knows $\hat{b}$ and ν.

Entrant, consequently, decides to enter or not the market and, in case of entry, chooses the optimal x_E of his product. A patent infringement occurs if $x_E < b$. Whenever there is an infringement, there will be a trial. If incumbent wins the trial, entry will not be permitted to the entrant (monopoly case), on the opposite case the patent will not be considered valid (price competition without patent protection).

Definition 2. *Incumbent's probability of winning the trial is $\mu(\nu, b)$. It is a continuous and differentiable function of ν and b, bounded between 0 and 1. It has the following properties:* $\lim_{b \to 0} \mu(\nu, b) = 1$, $\lim_{\nu \to 0} \mu(\nu, b) = 1$[3], $\frac{\partial \mu(\nu,b)}{\partial b} < 0$, $\frac{\partial \mu(\nu,b)}{\partial \nu} < 0$ *and* $\frac{\partial^2 \mu(\nu,b)}{\partial b \partial \nu} < 0$.

Whenever there is a trial both incumbent and entrant sustain administrative costs, respectively, C_I and C_E.

In the last stage, given the entry/quality decision and the outcome of the trial (if it takes place), price is formed in the market. Moreover, if entry takes place without infringement, or patent is invalidated after the infringement/validation challenge, there will be a price competition among the two firms. Otherwise only the monopolist will be in the market.

The strategic space is the following. Incumbent has a set A_I of infinite strategies,

[3] Since in Definition 1 we have that $\lim_{\nu \to 0} \hat{b} = 0$ and $b \leq \hat{b}$ by definition.

whose generic element $a_I \in A_I$ is defined below:

$$a_I = \{notpatent \times \{p_I\}, patent \times \{\nu, b\} \times \{p_I\}\}; \quad \nu > 0; \quad 0 < b \leq \hat{b}; \quad p_I > 0 \tag{2.3}$$

Clearly, *notpatent* and *patent* refer to incumbent's first stage decision. Given that he patents the invention, he chooses complexity and breadth $\{\nu, b\}$. Complexity is positive and the breadth is bounded between zero (not included) and $\hat{b}$ (which is below or equal to one). Finally, given entrant's strategy and the outcome of the trial – if it takes place – incumbent sets the price for its product.

Similarly, entrant has a set A_E of infinite strategies, whose generic element $a_E \in A_E$ is defined in expression n. 2.4.

$$a_E = \{notenter, enter \times \{x_E\} \times \{p_E\}\}; \quad 0 < x_E \leq 1; \quad p_E > 0 \tag{2.4}$$

Given incumbent's patenting decision, he then chooses to enter or not the market. If he decides to enter, sets the quality parameter x_E (given signals ν and $\hat{b}$). In the final stage, price competition will occur.

3 Pricing decision

In the last stage, wthe pricing decision depends on player 2's entry in the market. And if he enters, and there is a patent infringement ($x_E < b$), we must consider the outcome of the trial[4].

[4]With probability μ, the incumbent will win the trial, and entry for firm 2 will be denied by the court. With opposite probability, firm 2 will enter the market with quality level x_E.

If firm 2 does not enter the market or if firm 1 wins the trial, there will be a monopoly. Incumbent will, consequently, set $p_I = U$, which is the monopoly price.

In the opposite case – i.e. the entrant's choice is to enter the market with a certain product characteristic x_E, – given the characteristic chosen, both companies will compete in prices. In this stage, entrant's costs to develop the product (sustained to produce a certain level x_E) are sunk and, therefore, they are not considered in the optimisation problem.

Incumbent maximises the following profit function:

$$\pi_I = q_I\left(p_I\right)p_I = \left[\frac{p_E - p_I}{x_E}\right]p_I \tag{3.1}$$

Taking the first derivative of equation n. 3.1, we obtain the first order condition:

$$\frac{p_E - 2p_I}{x_E} = 0 \tag{3.2}$$

Entrant's demand is $q_E = 1 - q_I = \frac{x_E - p_E + p_I}{x_E}$. Therefore, he maximises the following expression:

$$\pi_E = q_E\left(p_E\right)p_E = \left[\frac{x_E - p_E + p_I}{x_E}\right]p_E \tag{3.3}$$

The first order condition is:

$$\frac{x_E - 2p_E + p_I}{x_E} = 0 \tag{3.4}$$

In the last stage, price competition results in the following equilibrium prices and quantities (expression n. 3.5):

$$\begin{aligned} &p_I^* = \tfrac{x_E}{3}; \quad p_E^* = \tfrac{2x_E}{3} \\ &\pi_I^* = \tfrac{x_E}{9}; \quad \pi_E^* = \tfrac{4x_E}{9} \end{aligned} \tag{3.5}$$

4 Entry and quality decision

Player 2's decision about entry and quality of the product depends on incumbent's patent decision in previous stage.

4.1 No patent protection

When incumbent decides not to patent its invention, the entrant has at his disposal two possible actions. Entering the market or not. The utility of the outside option is zero. While the unconstrained maximisation problem at this stage is:

$$\max_{x_E} \frac{4}{9} x_E - \frac{\beta}{2} x_E^2 \tag{4.1}$$

The first order condition is:

$$\frac{4}{9} - \beta x_E = 0 \iff x_E = \frac{4}{9\beta} \tag{4.2}$$

And the profit (equation n. 4.3) of entering is greater than zero. Trivially, when there is not patent protection the entrant will always enter the market.

$$\pi_E^U = \frac{8}{81\beta} > 0 \tag{4.3}$$

As in Yiannaka and Fulton [2006], we restrict the domain of β to be $\beta > \frac{4}{9}$, in order to have an interior solution ($x_E < 10$).

4.2 Patent protection

When the invention is patented, there will be asymmetric information about the protection granted. The incumbent can choose the patent breadth b in the interval $0 \leq b \leq \hat{b}$. E knows only the interval within which patent breadth is located (between 0 and $\hat{b}$).

Potential entrant, consequently, maximises its profit according to certain beliefs about the actual patent breadth. In this section we will derive the optimal quality for the entrant, given his beliefs about b.

For the Perfect Bayesian Equilibrium (PBE) we shall check beliefs' (weak) consistency according to the structure of the game. Given the information set reached, firm 2 *"should posit some single strategy combination which, in his view, has determined moves prior to his information set, and that his beliefs should be Bayes-consistent with this hypothesis"* [Kreps and Ramey, 1987].

Definition 3. *Potential entrant holds beliefs about the patent breadth equal to* $\tilde{b}$*, with* $0 < \tilde{b} \leq \hat{b}$*.*

Yiannaka and Fulton [2006] argue that the potential entrant has two types of strategies, when patent protection is granted. This is the result of uncertainty in the outcome of the trial, which occurs in case of infringement.

The first strategy consists in entering the market and avoid infringement, by setting an optimal quality (x_E) greater or equal to $\tilde{b}$. If beliefs are such that $\tilde{b} \leq 4/9\beta$, he will choose the unconstrained optimal quality, which corresponds to $x_E = 4/9\beta$ (From expression n. 4.2). Otherwise, the constrained solution leads to a quality $x_E = \tilde{b}$ [Yiannaka and Fulton, 2006].

Consequently, potential entrant may decide to avoid the trial by setting a quality parameter $x_E = \max\{4/9\beta; \tilde{b}\}$.

The second strategy is to enter the market with infringement, by setting a quality $x_E < \tilde{b}$. The optimal quality is set taking into account the uncertain outcome of the trial.

Firm 2 will maximise the following profit function:

$$\max_{x_E} (1-\mu)\frac{4}{9}x_E - \frac{\beta}{2}x_E^2 - C_E \tag{4.4}$$

With probability $1-\mu$ the patent is declared invalid and entry is permitted, otherwise firm 2 cannot enter the market. The problem in equation n. 11 has a solution for $x_E = (1-\mu)\frac{4}{9\beta} \leq \tilde{b}$.

Given the two possible strategies, we should check that beliefs are consistent with the game. One important remark should be done about the method to check beliefs. We consider weak consistency for the PBE. Therefore, we need beliefs to be consistent only in the equilibrium path. We demonstrate consistency by supposing that a certain strategy is the equilibrium strategy for firm 2 and by looking if beliefs are compatible with incumbent's behaviour. That is, given ν and $\hat{b}$, and beliefs $\tilde{b}$, there should not be an incentive for player 1 in deviating towards another $b \neq \tilde{b}$ ($b \leq \hat{b}$).

Property 1. *Suppose entrant decides always to avoid patent infringement, therefore beliefs are stable only if $\tilde{b} = \hat{b}$. If he decides always to seek the infringement challenge, beliefs converge to (but never reach) zero ($\tilde{b} \to 0$).*

Proof. Note that, at this point, we do not consider entrant's optimal strategy, but we check beliefs when entrant commits to a certain action.

Suppose the entrant has beliefs $\tilde{b} \leq \hat{b}$ and he commits to *avoid*, i.e. entrant chooses always a quality greater or equal to $\tilde{b}$. Beliefs are compatible with the game as long as:

$$\frac{\partial}{\partial \tilde{b}} \left[\frac{1}{9}\tilde{b} - \tau \right] = 0 \tag{4.5}$$

and

$$\frac{1}{9}\tilde{b} - \tau > \frac{4}{81\beta} \iff \tilde{b} > \frac{4}{9\beta} + 9\tau \tag{4.6}$$

Condition n. 4.5 means that the incumbent has no incentive in changing the breadth b, and beliefs are consistent with the game. While in condition n. 4.6, we are imposing that patenting the invention is more profitable than not patenting it, which is a necessary condition for incumbent's choice of patenting the innovation[5]. The equality in expression (4.5) is never satisfied, because it is always greater than zero. When the entrant chooses to avoid the infringement trial, for the incumbent is always

[5]Otherwise the information set would not be reached.

optimal to increase the patent breadth. Beliefs in this case are compatible with the structure of the game if and only if $\tilde{b} = \hat{b}$, because this is the maximum value of the patent breadth.

When the entrant does not avoid the infringement, consistency is verified if:

$$\begin{gathered} \frac{\partial}{\partial \tilde{b}} \left[\mu U + (1-\mu) \frac{4(1-\mu)}{81\beta} - C_I - \tau \right] = 0 \\ \frac{\partial \mu}{\partial \tilde{b}} U - (1-\mu) \frac{\partial \mu}{\partial \tilde{b}} \frac{8}{81\beta} = 0 \end{gathered} \tag{4.7}$$

and

$$\mu U + (1-\mu)^2 \frac{4}{81\beta} - C_I - \tau \geq \frac{4}{81\beta} \tag{4.8}$$

Condition n. 4.7 means that the incumbent has no incentive in changing the breadth, while n. 4.8 indicates that patenting the invention is more profitable than not patenting it. Since monopoly profits are bigger than industry profits in perfect competition[6], condition n. 4.7 is never satisfied, because the expression is always negative. When the entrant seeks the infringement trial, beliefs converge towards $\tilde{b} \to 0$. □

5 Characterisation of the game

In this section we will analyse the structure of the game, in order to characterise the Perfect Bayesian Equilibrium.

In the previous section we saw how beliefs – given ν (and $\hat{b}$) – change according to entrant's avoid/not avoid decision. Consequently, we shall consider, given beliefs,

[6] i.e. $U > \frac{8}{81\beta} + \frac{4}{81\beta}$, which are incumbent and entrant's profits with free entry and price competition

what is the most profitable strategy for the entrant.

Property 2. *Given patent complexity, entrant's profit function when he avoids the trial is non–increasing in* $\tilde{b}$*. Profits when the trial is not avoided are increasing in* $\tilde{b}$*.*

Proof. When E avoids the trial, he will choose a quality level $x_E = \max\left\{\frac{4}{9\beta}; \tilde{b}\right\}$, which leads to the following profit function:

$$\pi_E^{avoid} = \begin{cases} \frac{8}{81\beta} & \text{if } \frac{4}{9\beta} \geq \tilde{b} \\ \frac{4}{9}\tilde{b} - \frac{\beta}{2}\tilde{b}^2 & \text{otherwise} \end{cases} \tag{5.1}$$

The derivative of the profit function is negative ($\frac{\partial}{\partial \tilde{b}}\pi_E^{avoid} = \frac{4}{9} - \beta\tilde{b} < 0$) when $\tilde{b} > \frac{4}{9\beta}$ and zero otherwise. Thus, it is non–increasing in $\tilde{b}$.

When the entrant does not avoid the trial, the expected profit function is:

$$\pi_E^{navoid} = \frac{8}{81\beta}(1-\mu)^2 - C_E \tag{5.2}$$

Its derivative is $\frac{\partial}{\partial \tilde{b}}\pi_E^{navoid} = -\frac{16}{81\beta}(1-\mu)\frac{\partial \mu}{\partial \tilde{b}} > 0$ (since $\frac{\partial \mu}{\partial b} < 0$ by definition). π_E^{navoid} is, consequently, increasing in $\tilde{b}$. □

Property 3. *E's profit function when he avoids the trial does not depend on the complexity parameter* ν*. Profits when the trial is not avoided are increasing in* ν*.*

Proof. This is true because $\frac{\partial}{\partial \nu}\pi_E^{navoid} = -\frac{16}{81\beta}(1-\mu)\frac{\partial \mu}{\partial \nu} > 0$ (since $\frac{\partial \mu}{\partial \nu} < 0$), and $\frac{\partial}{\partial \nu}\pi_E^{avoid} = 0$ Complexity parameter affects only the probability of winning the trial, if it takes place. Consequently, when E avoids the trial there are no effects on his profit function. When infringement (and litigation) is not avoided, the greater is the complexity, the smaller is the incumbent's probability of winning the trial. It follows that the entrant's profit, when he does not avoid the trial, is increasing in ν. □

Definition 4. *Define $\rho(\nu)$ as the patent breadth, if it exists, which satisfies the following equality:*

$$\pi_E^{avoid}(\rho) = \pi_E^{navoid}(\rho, \nu) \tag{5.3}$$

It is implicitly defined by imposing the following equality (for $\rho > 4/9\beta$)

$$\tfrac{4}{9}\rho - \tfrac{\beta}{2}\rho^2 = \tfrac{8}{81\beta}\left(1 - \mu(\rho, \nu)\right)^2 - C_E \tag{5.4}$$

This parameter represents the indifference point between the two profit functions. When $x_E = \tilde{b} = \rho$, entrant is indifferent between avoiding or not patent infringement. In Appendix A, we demonstrate that a unique ρ always exists, although it could be greater than 1 (out of the range of our interest).

Property 4. *ρ is non–increasing in the complexity parameter ν, i.e. $\frac{\partial \rho}{\partial \nu} \leq 0$.*

Proof. ρ is defined as the value of b such that $\pi_E^{avoid}(\rho) = \pi_E^{navoid}(\rho, \nu)$. We can differentiate this equality for ν:

$$\tfrac{\partial}{\partial \rho}\pi_E^{avoid}(\rho)\tfrac{\partial \rho}{\partial \nu} - \tfrac{\partial}{\partial \rho}\pi_E^{navoid}(\rho, \nu)\tfrac{\partial \rho}{\partial \nu} - \tfrac{\partial}{\partial \nu}\pi_E^{navoid}(\rho, \nu) = 0 \tag{5.5}$$

For Property 2 and 3 we have:

$$\tfrac{\partial \rho}{\partial \nu} = \tfrac{\partial}{\partial \nu}\pi_E^{navoid}(\rho, \nu) \,/\, \left[\tfrac{\partial}{\partial \rho}\pi_E^{avoid}(\rho) - \tfrac{\partial}{\partial \nu}\pi_E^{navoid}(\rho, \nu)\right] \leq 0 \tag{5.6}$$

Because complexity increases entrant's expected profit from not avoiding the trial (Property 3), while π_E^{avoid} does not depend on complexity, increasing ν leads to a smaller ρ (a smaller indifference point between the two functions). □

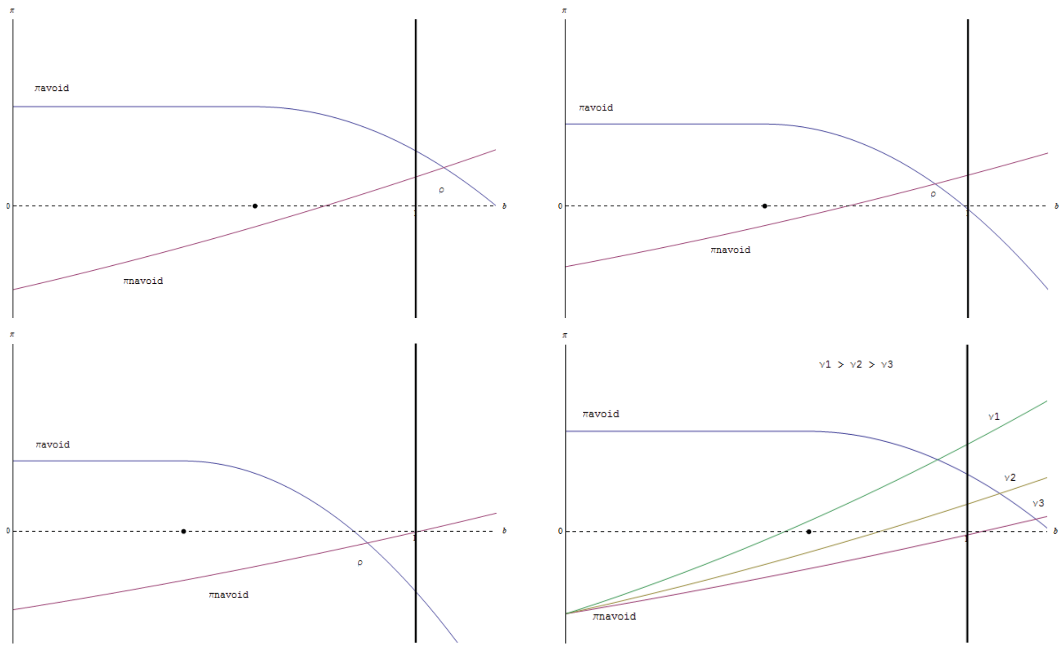

Figure 2: Profit functions and ρ given ν

In Figure 2, some possible cases are depicted. The top–left picture shows the case in which $\rho(\nu)$ is above one; in the top–right image, instead, ρ is below one. The bottom–left picture shows, the case in which both π_E^{avoid} and π_E^{navoid} are negative in ρ. Finally, last picture shows how π_E^{navoid} increases when complexity parameter increases ($\nu_1 > \nu_2 > \nu_3$).

Beliefs about the breadth are compatible with the game, if the entrant commits to the strategy "avoid", only when $\tilde{b} = \hat{b}$. On the other hand, when the trial is not avoided, beliefs converge towards (but never reach) zero. Moreover, we demonstrate that below ρ avoiding the trial is strictly preferred by the entrant. Putting together these considerations, we can define Proposition n. 1.

Proposition 1. *Given the patenting decision, and a complexity parameter $\nu > 0$. Entrant's beliefs about the actual patent breadth are:*

$$\tilde{b} = \begin{cases} \min\left\{\hat{b};\rho\right\} & if\ \hat{b} > \frac{4}{9\beta} \\ \forall b \in (0;\hat{b}] & otherwise \end{cases} \tag{5.7}$$

Where $\hat{b}$ depends on ν and is less than one, while ρ could be more than one.

Proof. When $\hat{b} \leq 4/9\beta$, every belief about the patent breadth is consistent with the game, because incumbent has not any incentive in changing patent breadth, since it does not affect his profit.

If $\hat{b} > 4/9\beta$, there are two cases to be considered. When $\hat{b} \leq \rho$, entrant finds more profitable to avoid the trial (his profit from the former strategy is bigger than the profit from not avoiding the trial for every breadth up to ρ). For every belief held ($\forall\tilde{b} \leq \hat{b}$), E will choose to avoid the trial and beliefs converge to $\hat{b}$. On the other hand, if $\hat{b} > \rho$, between zero and ρ entrant will prefer not to infringe. While, after this point, he strictly prefers not to avoid the trial. So, for $\forall\tilde{b} < \rho$, the entrant decides to avoid infringement, and for all $\tilde{b} \in (\rho;\hat{b}]$, he decides not to avoid the trial. For Property n. 1, beliefs converge towards ρ. Consequently, when $\hat{b} > 4/9\beta$, beliefs are consistent if and only if $\tilde{b} = \min\{\hat{b};\rho\}$. □

Given entrant's beliefs described in Proposition 1, the entrant will never find optimal to infringe the patent because beliefs are never above ρ.

Proposition 2. *Given beliefs $\tilde{b}$ defined in Proposition 1, entrant's optimal strategy will be:*

$$\begin{cases} \textit{enter and } x_E^* = \frac{4}{9\beta} & \textit{if } \hat{b} \leq \frac{4}{9\beta} \textit{ or the invention was not patented} \\ \textit{enter and } x_E^* = \tilde{b} & \textit{if } \hat{b} > \frac{4}{9\beta} \textit{ and } \frac{4}{9}\tilde{b} - \frac{\beta}{2}\tilde{b}^2 > 0 \\ \textit{not enter} & \textit{otherwise} \end{cases} \tag{5.8}$$

Proof. When $\hat{b} \leq 4/9\beta$, the optimisation problem is unconstrained and has a solution in $x_E^* = 4/9\beta$. The same result holds when incumbent has not patented the invention. When $\hat{b} > 4/9\beta$, E holds beliefs $\tilde{b} = \min\{\hat{b}; \rho\}$. For patent breadths smaller or equal to ρ, *not infringement* is weakly preferred to *infringement*. Thus, if profits $(\frac{4}{9}\tilde{b} - \frac{\beta}{2}\tilde{b}^2)$ are positive, entrant will choose a quality $x_E^* = \tilde{b}$. Otherwise, he will choose not to enter the market, since his profit is smaller than outside option's pay-off.

□

6 Patent complexity and breadth decision

Concealment of patent breadth has two effects. On the one hand, no matter what is the real patent breadth – and as long as the patent complexity is big enough to have $\hat{b} > 4/9\beta$ – entrant's beliefs will converge to $\min\left\{\hat{b}, \rho\right\}$. On the other hand, ρ fixes a limit for consistency of beliefs. This has a simple explanation, excessively high complexity is the result of a strategic behaviour aiming at lessening competition. E infers that the possible breadth is below that point, because chances of winning the trial for the incumbent decrease along with the increase in breadth and complexity.

Consequently, increasing complexity above the level such that $\hat{b} = \rho < 1$ has no effect on entrant's beliefs and strategies. Entry with infringement (and the trial) will never be chosen by the entrant, due to belief conformation[7]. Both patent breadth and patent complexity have the effect of decreasing incumbent's chances of winning the trial, if the trial takes place. I does not find optimal to choose a complexity parameter such that $\hat{b} > \rho$, because it increases only the chances of loosing the trial and does not lead to any gains, since it does not affect entrant's beliefs. Note that both $\hat{b}$ and ρ depend on ν. But, complexity affects these two parameters differently, $\hat{b}$ increases with ν, while ρ decreases.

Proposition 3. *If exists a ν^* such that $\pi_E^{avoid}(\hat{b}(\nu^*)) \leq 0$ and $\rho(\nu^*) \geq \hat{b}(\nu^*) > 4/9\beta$, then there exist at least one complexity parameter and an infinite number of patent breadths which deter entry in the market.*

Proof. Given the complexity parameter ν^*, entrant's beliefs are $\tilde{b} = \min\{\hat{b}(\nu^*), \rho(\nu^*)\}$ when $\hat{b}(\nu^*) > 4/9\beta$. Given that $\rho(\nu^*) \geq \hat{b}(\nu^*)$, entrant's beliefs are $\tilde{b} = \hat{b}(\nu^*)$. Due to monotonicity of the two profit functions, we have that between $\hat{b}(\nu^*)$ and $\rho(\nu^*)$ both pay-offs are negative. Thus, entrant will choose not to enter the market.

If $\pi_E^{avoid}(\hat{b}(\nu^*)) = 0$ and $\rho(\nu^*) \geq \hat{b}(\nu^*) > 4/9\beta$, ν^* is the smallest complexity parameter which deters entry.

If conditions of the proposition are met, infinite combinations of patent breadth and (at least one) patent complexity can deter entry: $\forall b \in (0, \hat{b}(\nu^*)]$ and ν^*. □

[7]We assume that, when entrant is indifferent between avoiding the trial or not, he chooses to avoid it

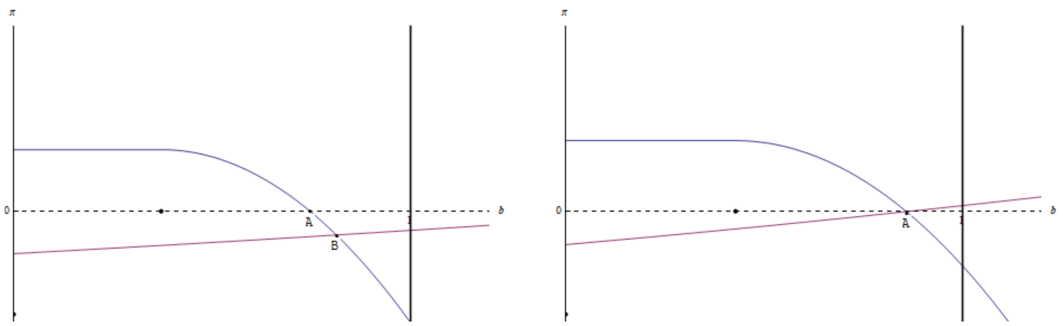

Figure 3: Entrant's profit function when entry can be deterred

Figure 3 shows player E's profit function. In the first figure, suppose $\hat{b}$ is somewhere between the two points A and B. Then $\hat{b}$ is less or equal to ρ (equality holds true in B), and entrant's beliefs about the patent breadth are equal to $\hat{b}$. Given these values, the expected profit function is negative and, consequently, incumbent deters entry of his competitor. The second figure depicts the case in which $\hat{b} = \rho$ and $\pi_E^{avoid}(\hat{b}) = \pi_E^{navoid}(\hat{b}) = 0$. Also in this case entry can be deterred.

Proposition 3 demonstrates that entry deterrence is possible when certain conditions hold true. This result is a direct consequence of beliefs conformation. As long as patent complexity is ν^*, and for every patent breadth $b \in (0, \hat{b}(\nu^*)$, beliefs held by the entrant are $\tilde{b} = \hat{b}(\nu^*)$[8].

Proposition 4. *When it is not possible to deter entry, as seen in Proposition n. 3. The incumbent finds optimal to set a complexity parameter $\dot{\nu}$ such that:*

$$\begin{aligned} \dot{\nu} &= \arg\max \hat{b}(\nu) \\ s.t.\ \hat{b}(\nu) &\leq \rho(\nu) \end{aligned} \tag{6.1}$$

[8]This value is below or equal to $\rho(\nu^*)$ in the proposition.

Proof. Incumbent's profit, when entry cannot be deterred, is increasing in x_E. E will set a quality parameter equal to $x_E = \tilde{b} = \min\{\hat{b}, \rho\}$ (Proposition n. 2). Consequently, I finds optimal to increase $\hat{b}$, up to the maximum point where $\hat{b}$ is below or equal to ρ. We can start from an initial complexity ν such that $\hat{b}(\nu) < \rho(\nu)$. Then entrant's beliefs about the patent breadth are $\tilde{b} = \hat{b}(\nu)$ and, consequently, he will enter the market[9] and choose product's quality $x_E = \tilde{b}$. Then incumbent can increase ν in order to increase $\hat{b}$. As consequence, ρ will decrease. The incumbent is willing to increase ν as long as $\hat{b}(\nu) \leq \rho(\nu)$. When $\hat{b}(\nu) = \rho(\nu)$ (if there exists a ν which satisfies the equality) further increases in the complexity of the patent reduce ρ and, therefore, entrant's beliefs about the breadth. So it is not optimal to increase complexity up to the point where $\hat{b} = \rho$.

□

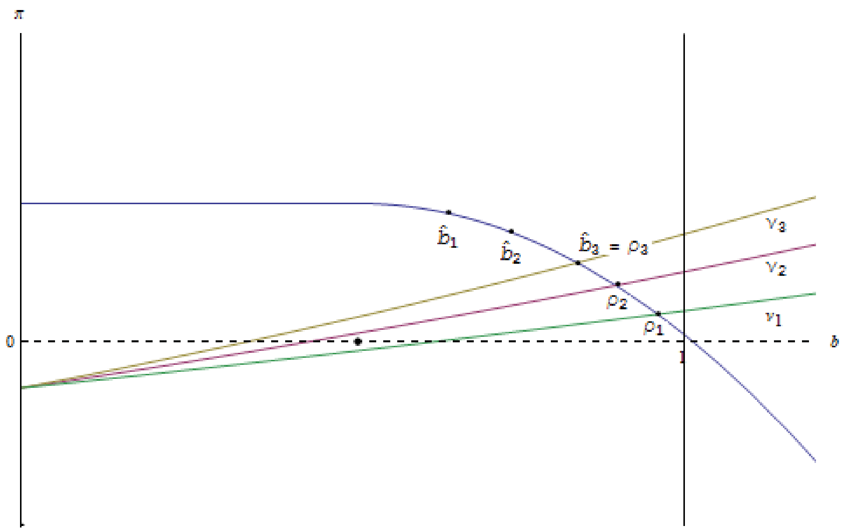

Figure 4: Entrant's profit function when entry cannot be deterred

[9]We assume entry cannot be deterred

7 Patent decision

Given the setting characterised in the previous pages and the definition of beliefs, we can derive the Perfect Bayesian Equilibrium of the game.

Proposition 5. *A Perfect Bayesian Equilibrium in the game consists in a tuple of strategies* (a_I^*, a_E^*) *such that:*

$$a_I^* = \begin{cases} \textit{patent and set } \nu = \nu^+ \textit{ such that entry is avoided and } p_I = U \textit{ and } b = \forall b \leq \hat{b}(\nu^+) & \textit{if } \nu^+ \textit{ exists} \\ \textit{patent and set } \nu = \nu^* \textit{ and } b = \forall b \leq \hat{b}(\nu^*) \textit{ and } p_I = \frac{1}{3} x_E^* & \textit{if } \frac{1}{9}\hat{b} - \tau \geq \frac{4}{81\beta} \\ \textit{not patent and } p_I = \frac{1}{3} x_E^* & \textit{otherwise} \end{cases}$$

$$\textit{where } \nu^* = \arg\max \hat{b}(\nu) \textit{ s.t. } \hat{b}(\nu) \leq \rho(\nu) \tag{7.1}$$

$$a_E^* = \begin{cases} \textit{enter and } x_E^* = \frac{4}{9\beta}, p_E = \frac{2}{3} x_E^* & \textit{if } \hat{b} \leq \frac{4}{9\beta} \textit{ or the invention was not patented} \\ \textit{enter and } x_E^* = \tilde{b}, p_E = \frac{2}{3} x_E^* & \textit{if } \hat{b} > \frac{4}{9\beta} \textit{ and } \frac{4}{9}\tilde{b} - \frac{\beta}{2}\tilde{b}^2 > 0 \\ \textit{not enter} & \textit{otherwise} \end{cases} \tag{7.2}$$

Where $\tilde{b}$ *denotes beliefs held by the potential entrant (defined in Proposition n. 1).*

Proof. Under patent protection, entrant's beliefs about the breadth converge to the minimum value between $\hat{b}$ and ρ, regardless to the real breadth. When entry deterrence is feasible, Incumbent will choose to pursue this strategy. Otherwise, the optimal strategy consists in hiking the effect of concealment, which has a positive impact on incumbent's profits, so he sets the maximum value of $\hat{b}$ under the constraint of being smaller or equal to ρ. Infringing the patent is never optimal for player 2, given his beliefs. Consequently, he sets a quality parameter greater or equal to $\tilde{b}$, if his profit is positive. Otherwise, he will not enter.

I decides to patent its invention or not. Due to administrative costs, needed to patent the invention, the decision *not to patent* will be based on comparison of the two profits. When $\tilde{b}$ is below the optimal unconstrained x_E $(4/9\beta)$, the incumbent does not patent, because patent decision does not affect the optimal x_E chosen by the entrant and I sustains an administrative cost to fill the patent application. Patent the invention is profitable – if entry cannot be deterred – when $\hat{b}(\nu^*) \geq \frac{4}{9\beta} + 9\tau$. □

Proposition 6. *Unless the Incumbent does not find optimal to patent the invention, there exists an infinite number of Perfect Bayesian Equilibria in the game.*

Proof. Since beliefs depend only on ν, incumbent's best strategy (when he patents the product) can be implemented by a single complexity parameter and all patent breadths $b \leq \hat{b}$. Thus, equilibria are infinite. Moreover, when entry can be deterred (Proposition n. 3), and there are more than one complexity parameter which can deter entry[10], the incumbent is indifferent between one of these parameters. Consequently, PBE is not unique.

□

8 Conclusions

Economic literature extensively explores the role of asymmetric information and patent protection in anticompetitive behaviours. These two aspects, however, have

[10]This happens when there exists a ν^* such that $\pi_E^{avoid}\left(\hat{b}(\nu^*)\right) = 0$ and $\hat{b}(\nu^*) < \rho(\nu^*)$, the strict inequality means that more than one complexity parameter can deter entry.

never been analysed jointly. This paper analyses if asymmetric information can be used in the patent process to lessening competition.

To do so, we introduce the concept of patent complexity, which, in our fashion, is closely related to patent breadth. Complexity is linked both to the structure of a certain product/invention (since it is reflection of complexity in the production of inventions) and to information which spills over to potential competitor about the invention itself. We put a great deal of attention on the last one, to stress the role of information in patent process. Thus, we adapt the model of Yiannaka and Fulton [2006] of patent (breadth) decision on a product with drastic innovation.

Drastic innovation is chosen, because it leads to more uncertainty about the protection which should be granted by mean of the patent. Uncertainty affects also potential entrants' beliefs on protection granted. Moreover, with drastic innovation tPatent offices' ability in assessing the patent could be limited, and the efficient patent system hypothesis is more likely not to hold.

We find that patent complexity, as defined in the model, can deter entry for certain parameters' values. When entry cannot be deterred, complexity is used to increase incumbent's profit. As in Yiannaka and Fulton [2006, 2011], we find that, when entry cannot be deterred, there is a greater product differentiation between the two players' product (because x_E is at least equal to the unconstrained solution).

Some final remarks should be done. We assume entrant is not able to recognise the boundaries of protection granted by the patent (i.e. the patent breadth), but he only infers the interval within which the breadth is located. The more is the complexity the more difficult is identify the real patent breadth, because this interval becomes wider. Therefore, we are assuming bounded rationality due to asymmetric information.

However, a great degree of could be the result only of a strategic behaviour, not reflected the actual breadth, this results in an upperbound for beliefs, which is ρ. Because the actual breadth is not known, the incumbent may set a great complexity and a low breadth, so to maximise the expected value from infringement. As a result of entrant's beliefs inference process, however, this strategy is not considered credible. Asymmetric information has two effects: on the one hand the incumbent increases complexity in order to increase E's costs of entry; on the other hand, it limits the set of possible actions because of credibility problems.

The model underlines the competitive threats of information in the patenting system, especially in those sectors (such as ICT) in which the degree of innovation is really high. The asymmetries on the information about the protection granted could be used as entry deterrence instrument, because competitors are discouraged by the threat of the trial. This could be compatible with some examples, such as Microsoft's claim of patent infringement by Linux[11], in which the complexity of the patent does not permit to understand the actual patent breadth, therefore a settlement is preferred to the threat of the trial.

Two major limitations of the model are that there is always a trial if there is infringement, and that in equilibrium there is not a trial. The first assumption is the same of Yiannaka and Fulton [2006] and is needed in order to maintain the model

[11] see for instance *MSFT: Linux, free software, infringe 235 of our patents*, CNN Money: http://features.blogs.fortune.cnn.com/2007/05/13/msft-linux-free-software-infringe-235-of-our-patents/

mathematically treatable. The second second result may not reflect reality, although Microsoft example above could be an evidence that firms may be willing to sustain extra–costs in order to avoid the trial. In our model these costs are represented by the greater deal of differentiation.

Appendix

A Existence of ρ

Define ρ as the positive number, if it exists, such that the potential entrant is indifferent between avoiding and not avoiding patent infringement. It results from the following equality:

$$\pi_E^{avoid}(\rho) = \pi_E^{navoid}(\rho, \nu) \tag{A.1}$$

Profits when infringement is not avoided are:

$$\pi_E^{navoid}(\rho) = \frac{8}{81\beta}\left(1 - \mu(\rho, \nu)\right)^2 - C_E \tag{A.2}$$

Since $\mu(\rho, \nu)$ is a probability (thus bounded between 0 and 1), we have that π_E^{navoid} is also bounded between $\frac{8}{81\beta} - C_E$ and $-C_E$.

Therefore the equality never holds when $\rho \leq 4/9\beta$, because, in this interval, $\pi_E^{avoid} = 8/81\beta$; which is strictly greater than the former profit function. Consequently, this equality should be considered for $\rho > 4/9\beta$.

$$\frac{4}{9}\rho - \frac{\beta}{2}\rho^2 = \frac{8}{81\beta}\left(1 - \mu(\rho, \nu)\right)^2 - C_E \tag{A.3}$$

To demonstrate existence, we can check if, for certain values of ρ, π_E^{avoid} is above $\frac{8}{81\beta} - C_E$ (the upperbound of π_E^{navoid}) and, for others, is below $-C_E$ (lowerbound). By continuity of both functions, we can argue that the equality holds in (at least) one point. By monotonicity we have that equality holds in only one point.

For $\rho \leq 4/9\beta$, as we said, $\pi_E^{avoid} = 8/81\beta > \frac{8}{81\beta} - C_E$. While π_E^{avoid} is below $-C_E$ when:

$$\frac{4}{9}\rho - \frac{\beta}{2}\rho^2 < -C_E \quad \Longleftrightarrow \quad \rho > \frac{1}{\beta}\left[\frac{4}{9} \pm \sqrt{\frac{16}{81} + 2\beta C_E}\right] \cap \rho > 4/9\beta \tag{A.4}$$

Then – since expression (A.4) should be considered for $\rho > 4/9\beta$ – for $\rho > \frac{1}{\beta}\left[\frac{4}{9} + \sqrt{\frac{16}{81} + 2\beta C_E}\right]$ we see that $\pi_E^{avoid} < \pi_E^{navoid}$. Therefore, ρ exists and is unique.

Note that ρ could be greater than 1, but it always exists. Figure n. 5 represents graphically the proof.

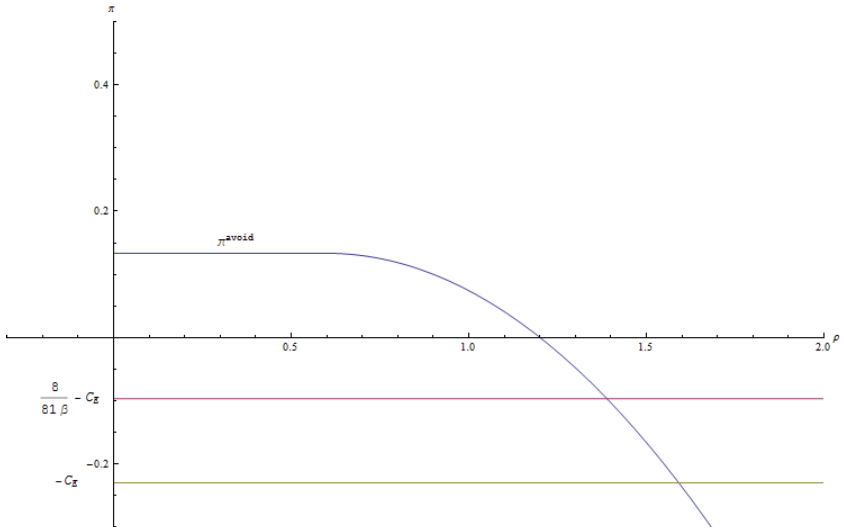

Figure 5: Existence of ρ

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Chapter 3

Who hires innovative artists?

Abstract

This paper investigates the selection of artists by a gallery with adverse selection and moral hazard. Artists have heterogeneous productivity and are allocated into two groups: those who innovated and those who did not. Artists reveal their type by participating to an auction where the employer offers a menu of contracts specifying output and wage. When the gallery has monopolistic power, price is set to give a premium to innovation. Thus, in a Bayesian-Nash equilibrium, choosing between two low productive artists, the gallery hires the artist who innovated. If, however, the gallery has little market power then it prefers, between two low productive artists, the one who did not innovate because her output is closer to the first-best solution. This result indicates that a segmented market with gate-keeping, where some artists have no opportunity to bid to join a top gallery, has a negative impact on innovation.

JEL Classification: D82; D86; Z11; C72; L14

1 Introduction

We analyse a common circumstance in the art market, namely that of a gallery manager searching for 'new' interesting artists to show. That manager is likely to take into account the indications of influential curators, institution directors or collectors about what is innovative in the art market. However, when he decides to pick an artist who had her first breakthrough in the art scene, he faces the problem of defining an agreement (contract) with the artist in conditions of uncertainty. Common uncertainty for the parts of the contract derives from market demand. We overlook this problem to focus instead on the uncertainty that the manager has about the intrinsic productive abilities of the artists and the effort supplied by the artist for the success of the show, which are both private information of the artist. Our study presents an analytical model investigating the definition of an optimal contract to offer to artists that differ for having produced innovative work and for their creativity, interpreted here as artistic productivity and ability of being innovative in the future. The model then includes both adverse selection (the hidden information about creativity) as well as moral hazard (the hidden effort of the artist). Section 2 describes the behavioural underpinnings of the model and explains the analytical results derived in the section that follows. Section 3 defines the model and presents some preliminary results. Section 4 concludes the paper with some comments on the implications of the results and describes future extensions of the analysis.

2 Contracts for creative artists with hidden information and action

In this paper we focus exclusively on visual artists and on the optimal contracts that galleries may offer to those artists when their creativity and efforts are private information. As indicated by Menger (2006), contemporary artists tend to be ranked according to their creative artistic talent rather skills. There is a substantial multidisciplinary literature dealing with creativity and innovation, although they are not always addressed distinctly. These issues, and their determinants, have been analysed referring to individuals (Simon, 1967), to the public (Frey, 2002) or to organizations (King and Anderson, 1995; Castañer and Campos, 2002). They were also applied to different activities, which further qualify the nature of creativity and innovation. It is rather surprising to notice that, even though creativity is widely believed to be the essence of artistic production[1], the economic analysis of creativity is rather limited (as indicated by Throsby, 2001; Towse, 2006). It is beyond the scope of this paper trying to summarize the main findings of research.

We aim at contributing to the literature by investigating the optimal contract problem between an artist and a gallery that wishes to show her work. The analysis envisages a typical situation in the art market where a gallery manager is not informed either about intrinsic characteristics of the artist, such as creativity, or the effort that the artist will invest in her production after signing a contract[2]. However,

[1] Creativity is often interpreted as an artistic mode of production (Zukin, 1982; Caves, 2000).

[2] An overview of the characteristics of contracts for creative products is provided by Caves (2000). In our model we simply refer to a contract for a show or for a fixed number of shows.

the manager knows previous work of the artist and particularly whether this has been recognized as innovative or not.

This problem of combined hidden information and action is central in the activity of galleries that constantly recruit new artists for their yearly season. Yet this issue has never been studied analytically, to the best of our knowledge. The closest study is Cellini and Cuccia (2003) that investigates a repeated game where a private financier has imperfect information on the preferences of an artist for experimental production vs. commercial production and wants to finance only the latter whereas the artist may prefer to experiment.

Here we adopt very simple and specific definitions of creativity and innovation, which are nonetheless rather established in the literature. Creativity is defined as the long-lasting ability of an artist to set goals and to solve the problems hindering their achievement in ways that revel novel thinking (Simon, 1967). In line with human capital theory, we assume that higher artistic ability implies higher productivity for a given level of effort (Towse, 2006). In addition it reduces the cost of artistic production, it is reasonable to expect that creativity has an economic value and it is sought by art buyers, because it expresses the artist's ability to produce novel work and, therefore, increases the likelihood that the artist will survive in the market and be a good investment. A realistic assumption is that creativity is private information of the artist.

Innovation, on the other hand, is a more problematic issue at least because it may concern rather science than art. We interpret innovation as a single breakthrough of an artist. Clearly there are distinctions to make on the relative scale of innovation.

At its peak, innovation in the arts can lead to a new 'artistic language' that will be embodied in the work of other artists. For the sake of this paper, we adopt a more restricted sense and consider innovative a work acclaimed as original or non-conventional by curators. We avoid dealing with the itchy subject of recognizing innovation and priority in the arts (Caves, 2000; Menger, 2006), by assuming that there is an established system of gate-keepers (curators, editors, institutions, ...) who 'certify' innovation and that this is prized by the market, for the obvious reason that distinguishes an artist from her peers and provides monopoly power[3].

A common tenet seems to be that creativity is a precondition to innovation (Bassett-Jones, 2005). In this respect, creativity represents the fertile soil for innovation or, in other words, the artist's ability of being innovative.

In conclusion, in the model presented in the next section innovation differs form creativity because it has been recognized as such and then is known to all the actors in the art market. On the contrary, creativity is not revealed to the gallery manager and the buyers before the work is completed (it is private information of the artist).

For the reasons explained previously, both creativity and innovation have a positive impact on demand. The fact that an artist has been innovative does not guarantee that she is creative, as this innovation could have been a fortuitous event[4].

[3]It is common wisdom that artists selected for the most important stages for innovative artworks such as Venice Biennale or Documenta has a positive impact on their sales often visible in Art Basel, the most important art fair in the world, which takes place a few days before the openings of those important events. However, some art buyers may not praise innovative art and be attracted by more conventional production. The hypothesis of innovation harmful for sales is addressed by Cellini and Cuccia (2003).

[4]One could argue that the manager may wait for a second innovation, before hiring the artist. It should be noticed that several prominent artists built their own fame just on one path-breaking innovation, although few others were indeed able to innovate on their previous work. Moreover, it should be noticed that the market does not always appreciate novelties, in the sense that artists

However an artist who has innovated is realistically more likely to be creative than an artist who has never innovated. A gallery manager considering the option of offering a show to an artist knows whether the latter has innovated or not (as recognized by gatekeepers), but faces uncertain profits since does not know the productivity (creativity) of the artist and the effort she will put in the artistic production[5]. This problem is solved by the manager by offering an optimal scheme inducing truth telling, which links the probability of being hired to wage and the quantity produced.

Results show that an innovative artist who is also creative will be selected with certainty. Interestingly an artist who is not creative, but who has been recognized as innovative, may be selected against creative artists who has not yet been recognized as an innovative one. This happens when the impact of innovation on sale price is sufficiently higher than the impact of creativity on price. In particular, the difference between the two impacts should more than compensate the distortion caused by choosing the non creative artist.

This result suggest that when a gallery is able to command a very high premium on innovation an artist will be selected also if she has low productivity. On the contrary, when the gallery is unable to charge high prices for the work of artists who have been declared innovative (for example because of low market power), and it faces two non creative artists, we have the surprising result that the gallery may select the artist who did not innovate. The reason is that the innovation premium is not able to compensate for the distortion from imperfect information about creativity.

may end locked in doing their 'typical' work because there is not a sufficient demand for novel (or atypical) work.

[5] In our model we do not address quality outside of innovation. Therefore low (high) effort leads to low (high) production.

3 The Model

There are two groups of artists, those who are recognised as innovative by gatekeepers and those who are not. Furthermore, artists can differ for their level of creativity, we assume that an artist can have high or low creativity.

Innovation is a public information, while the creativity is a private information not known by the Gallery.

We indicate with $\overline{\theta}$ the productivity parameter for the high type, and with $\underline{\theta}$ the creativity parameter for the low type. Creativity parameters are such that $\overline{\theta} > \underline{\theta} > 1$, the latter inequality is needed to avoid negative optimal quantities.

In the two groups there are both low and high creativity workers. However, they differ for the proportion of high–type artists. Define k as the percentage of high–creativity artists in the group of innovators, and r to be the proportion of high–type artists in the group of non–innovators. We assume that $k > r$, which means that it is more likely to have a high–creativity worker among innovators rather than among those who have not innovated.

The price paid by consumers for art items is defined as follows:

$$p_j = 1 + \hat{i}_j + \alpha(\theta_j) - \frac{\beta}{2} y_j \tag{3.1}$$

Where y_j represents the number of artworks created by the selected artist j, and parameters $\hat{i}$ and $\alpha(\theta_j)$ are price parameters which depend on the selected artist's characteristics. We assume that $\beta < 1/4$ and, to assure interior solutions.

We assume that consumers are willing to pay a higher price to innovators, by setting:

$$\hat{i}_j = \begin{cases} i \geq 0 & \text{if } j \text{ is an innovator} \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

Furthermore, we assume that the more creative is the artist, the more he is appreciated by customers, therefore:

$$\alpha(\theta_j) = \begin{cases} \alpha \geq 0 & \text{if } j \text{ is a high–creativity artist} \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

Parameters i and α should be interpreted as differentiation parameters with respect to the two groups and the two types of artists. We are assuming that previous innovation and the creativity of an artist are distinctive qualities which are recognised by consumers, and which increase their willingness to pay.

The quantity of artistic items produced is a continuous variable; it is equal to $y_j = \theta_j + e_j$, where e_j indicates the level of effort exerted by the artist, and θ_j represents his creativity. Since the creativity of an artist is not publicly observable, it is not possible to determine the level of the effort.

To produce a certain effort level, we assume that a generic artist has a cost function such that:

$$C(e_j, \theta_j) = \tfrac{e}{\theta_j}; \quad \theta_j \in \{\underline{\theta}, \overline{\theta}\} \tag{3.4}$$

The cost function is decreasing in the creativity parameter, *i.e.* $\partial C(e)/\partial\theta < 0$. As consequence, the high–creativity type sustains smaller costs to exert a certain effort than those that the low–creativity type would sustain with the same level of effort. Furthermore, the cost function can be expressed as follows:

$$C(y_j, \theta_j) = \frac{(y_j - \theta_j)}{\theta_j}; \quad \theta_j \in \{\underline{\theta}, \overline{\theta}\} \tag{3.5}$$

The model is structured as follows. Nature selects two candidates, one among innovators and one in the remaining group. The two selected artists can have high or low creativity, according to *ex–ante* probabilities k and r. The gallery, then, will choose only one artist between the two candidates.

Creativity is a private information, which is not known by the gallery. The group from which an artist comes from, instead, is a publicly observable information.

This model is similar to Laffont and Tirole (1993) and Kübler (1997). We, however, allow for the possibility that the price is influenced by the characteristics of the artist. Furthermore, the quantity produced influences the price.

In our opinion, this could fit more accurately the art market. Indeed the characteristics of an artist – both his intrinsic characteristics (here defined as creativity) and his past experience (being innovative, having experimented in the past) – affect the price of his creations. As well as the number of creations available in the market affects the price.

In next section, we will consider the first best optimum, which corresponds to the case in which creativity is publicly observable. In the following pages we will consider a optimal mechanism with truth telling, which implements the second best optimum for the Gallery. It consists in a menu of task assigned, wage and probability of being hired.

4 Perfect Information Benchmark

As mentioned before, two candidates – one for each group – are selected by Nature among the population of artists. Then, the Gallery hires only one of them.

Depending on the move of nature, there are four cases to be considered, which differ for the types of artists selected in the two groups (See Table n.1)

Case	Innovator	Non–Innovator	Probability
A	$\overline{\theta}$	$\overline{\theta}$	kr
B	$\overline{\theta}$	$\underline{\theta}$	$k(1-r)$
C	$\underline{\theta}$	$\underline{\theta}$	$(1-k)(1-r)$
D	$\underline{\theta}$	$\overline{\theta}$	$(1-k)r$

Table 1: Four cases

Given that an artist j is selected (between the two candidates I and NI), optimum is reached by assigning – to the artist j – the quantity y_j which maximises:

$$\max_{j \in \{I,NI\};y_j} \left[1 + \hat{i}_j + \alpha(\theta_j) - \frac{\beta}{2} y_j\right] y_j - C(y_j, \theta_j) \tag{4.1}$$

The Gallery selects only one artist and commissions a quantity of artworks y_j.

Proposition 1. *With complete information about artists, given that the selected artist is j, the optimal quantity of art commissioned, the optimal wage and optimal Gallery's profits are:*

$$y_j^*(\theta_j) = \tfrac{1}{\beta}\left[1 + \hat{i}_j + \alpha(\theta_j) - \tfrac{1}{\theta_j}\right]; \quad w_j^* = \tfrac{y_j^* - \theta_j}{\theta_j}; \pi^* = 1 + \tfrac{1}{2\beta}\left[1 + \hat{i}_j + \alpha(\theta_j) - \tfrac{1}{\theta_j}\right]^2 \tag{4.2}$$

The above result derives from the maximisation problem in expression (4.1) (See

Appendix A for derivations). The Gallery pays to the selected artist a wage equal to the cost sustained to provide y_j^*. The optimal quantity of art creations is increasing in the creativity parameter θ_j, also Gallery's profits are an increasing function of θ_j. Gallery's profits, the number of artworks and artist's wage are increasing in the innovation parameter i (See Appendix B). The effect of θ_j on wages depends on model parameters.

Given the above considerations, if the two artists have the same creativity, the innovator is preferred by the Gallery.

Therefore, in cases A, B and C in Table 1, first best is reached by selecting the artist who has innovated; because he leads to greater profits than those which would be achieved by hiring the non innovator. This is a combination of both a different willingness to pay of consumers, and (in case B) of smaller marginal costs of the innovator (who is highly creative). In case D, the choice between the two artists depends on model parameters.

Proposition 2. *With complete information about artists' capabilities, the Gallery will always select the artist who has innovated when he is highly creative. The Gallery will select the low–creativity innovator always if the other candidate is a low–creativity artist. When the other candidate is a highly–creative artist, the Gallery will select the low–creative innovator if and only if* $i - \alpha \geq \frac{1}{\underline{\theta}} - \frac{1}{\bar{\theta}}$.

This proposition summarises the above considerations. In this model we must consider both the differences in productivity, and the effect on prices of personal characteristics of artists. Therefore, with perfect information, we can define the probability of being hired of artists, in the four different cases[6]:

$$P_I^{fb}(\overline{\theta},\overline{\theta}) = P_I^{fb}(\overline{\theta},\underline{\theta})P_I^{fb}(\underline{\theta},\underline{\theta}) = 1; \qquad P_I^{fb}(\underline{\theta},\overline{\theta}) = \begin{cases} 1; & \text{if } i \geq \alpha + \Delta \\ \\ 0 & \text{otherwise} \end{cases}$$

$$P_{NI}^{fb}(\overline{\theta},\overline{\theta}) = P_{NI}^{fb}(\overline{\theta},\underline{\theta}) = P_{NI}^{fb}(\underline{\theta},\overline{\theta}) = 0; \quad P_{NI}^{fb}(\underline{\theta},\overline{\theta}) = \begin{cases} 0 & \text{if } i \geq \alpha + \Delta \\ \\ 1 & \text{otherwise} \end{cases} \tag{4.3}$$

The innovator is always hired when he is competing with the non–innovative low–creativity artist, or when he is highly creative (regardless to the characteristics of the other candidate). Due to the effect on profits of i, α and creativity parameters, the non innovative – but high creative – artist can be selected when i is small enough. The Gallery selects, among the two candidates, the artist who leads to the highest profits (as defined in Proposition 1)

5 Market Solution

In this section we consider an optimal direct mechanism solution with truth telling. We use a similar environment to Laffont and Tirole (1993) and Kübler (1997). In our setting, however, the price is a decreasing function of the quantity produced, and

[6]Note that we set $\Delta = \frac{1}{\underline{\theta}} - \frac{1}{\overline{\theta}}$.

it also depends on selected artist characteristics.

The mechanism is constituted by probabilities of being hired by the Gallery – $P_I(\theta_I, \theta_{NI})$ and $P_{NI}(\theta_I, \theta_{NI})$ – by non negative wages $w_I(\theta_I, \theta_{NI})$ and $w_{NI}(\theta_I, \theta_{NI})$, and by quantities of art to be produced $y_I(\theta_I, \theta_{NI})$ and $y_{NI}(\theta_I, \theta_{NI})$.

The Gallery will maximise the expected profit function shown in Expression (C.1) in Appendix C, with respect to probabilities of hiring an innovator or a non-innovator (in the four cases), wages and quantities of art commissioned. The problem is a constrained maximisation, because the optimal choice must consider incentive compatibility constraints and individual rationality constraints of the two types of artist, and of the two groups of worker.

Proposition 3. *The Gallery assigns to the highly creative artists tasks which are equal to the first best optimum. The low productive artists produce a quantity which is lower than the optimal one with complete information. Equilibrium quantities are shown below.*

$$\begin{aligned} y_I(\overline{\theta}) &= \tfrac{1}{\beta}\left[1 + i + \alpha - \tfrac{1}{\overline{\theta}}\right]; \quad y_I(\underline{\theta}) = \tfrac{1}{\beta}\left[1 + i - \tfrac{1}{\underline{\theta}} - \tfrac{k}{1-k}\Delta\right]; \\ y_{NI}(\overline{\theta}) &= \tfrac{1}{\beta}\left[1 + \alpha - \tfrac{1}{\overline{\theta}}\right]; \quad y_{NI}(\underline{\theta}) = \tfrac{1}{\beta}\left[1 - \tfrac{1}{\underline{\theta}} - \tfrac{r}{1-r}\Delta\right]; \end{aligned} \tag{5.1}$$

The task assigned to the high–creativity artist is equal to the first best optimum (Proposition 1), while there exists a distortion – with respect to the perfect information case – for the low creativity type. This distortion depends on the proportion if high creativity artist in the group – *i.e.* r for innovators, and k for non–innovators – and on the creativity parameters, since $\Delta = \frac{1}{\underline{\theta}} - \frac{1}{\overline{\theta}}$. Moreover, this distortion is higher for innovators. This has some distorting effects on probabilities of being hired, as we can see in the following proposition.

Proposition 4. *Under the optimal contract the high–creativity innovator is always hired. The low creativity innovator may not be hired, depending on parameters, both when the non innovator has high and low creativity. (See Table 2)*

$P_I(\bar{\theta},\bar{\theta})$	1	1	1
$P_I(\bar{\theta},\underline{\theta})$	1	1	1
$P_I(\underline{\theta},\underline{\theta})$	0	1	1
$P_I(\underline{\theta},\bar{\theta})$	0	0	1
	$i < \frac{k-r}{(1-k)(1-r)}\Delta$	$\frac{k-r}{(1-k)(1-r)}\Delta < i < \alpha + \frac{1}{(1-k)}\Delta$	$i > \alpha + \frac{1}{(1-k)}\Delta$
	$u_2 < u_4$	$u_2 < u_3$	$u_2 > u_3$
	$u_2 < u_3$	$u_2 > u_4$	$u_2 > u_4$

Table 2: Three Solutions with respect to i

Note, furthermore, that it always holds that $P_I(\theta_I,\theta_{NI}) + P_{NI}(\theta_I,\theta_{NI}) = 1$.

The innovator with low creativity may not be hired when it is first best optimal to hire him, since probabilities are different with respect to the optimal benchmark. This is mainly due to the distortion on the quantity commissioned to low creativity artists.

In particular, even if in first best is always optimal to hire the innovator when both artist are low creative – *i.e.* $P_I^{fb}(\underline{\theta},\underline{\theta}) = 1$ always – for enough small values of i, the (positive) price effect of choosing innovators is overcome by the (negative) effect due to task distortion, leading to smaller profits.

For the same reason, the chances of hiring a high–creativity non innovator increase when the innovator increase is low creative. The result is that only for certain values of parameters the market solution has the same probabilities of the perfect information benchmark (second and forth columns in Table 3).

$P_I^{fb}(\underline{\theta},\underline{\theta})$	1	1	1	1
$P_I^{fb}(\underline{\theta},\overline{\theta})$	0	0	1	1
$P_I(\underline{\theta},\underline{\theta})$	0	1	1	1
$P_I(\underline{\theta},\overline{\theta})$	0	0	0	1
	$i < \frac{k-r}{(1-k)(1-r)}\Delta$	$\frac{k-r}{(1-k)(1-r)}\Delta < i < \alpha+\Delta$ *	$\alpha+\Delta < i < \alpha + \frac{1}{(1-k)}\Delta$	$i > \alpha + \frac{1}{(1-k)}\Delta$ *

Table 3: Comparing solutions with Perfect Information Benchmark

Proposition 5. *Under the optimal contract, given the equilibrium probabilities, wages are*

$$w_I(\overline{\theta},\overline{\theta}) = \frac{y_I(\overline{\theta})-\overline{\theta}}{\overline{\theta}} + P_I(\underline{\theta},\overline{\theta})y_I(\underline{\theta})\Delta; \qquad w_I(\overline{\theta},\underline{\theta}) = \frac{y_I(\overline{\theta})-\overline{\theta}}{\overline{\theta}} + P_I(\underline{\theta},\underline{\theta})y_I(\underline{\theta})\Delta;$$

$$w_I(\underline{\theta},\overline{\theta}) = P_I(\underline{\theta},\overline{\theta})\frac{y_I(\underline{\theta})-\underline{\theta}}{\underline{\theta}}; \qquad w_I(\underline{\theta},\underline{\theta}) = P_I(\underline{\theta},\underline{\theta})\frac{y_I(\underline{\theta})-\underline{\theta}}{\underline{\theta}};$$

$$w_{NI}(\overline{\theta},\overline{\theta}) = 0; \qquad w_{NI}(\overline{\theta},\underline{\theta}) = 0;$$

$$w_{NI}(\underline{\theta},\overline{\theta}) = P_{NI}(\underline{\theta},\overline{\theta})\frac{y_{NI}(\overline{\theta})-\overline{\theta}}{\overline{\theta}} + P_{NI}(\underline{\theta},\underline{\theta})y_{NI}(\underline{\theta})\Delta; \qquad w_{NI}(\underline{\theta},\underline{\theta}) = P_{NI}(\underline{\theta},\underline{\theta})\frac{y_{NI}(\underline{\theta})-\underline{\theta}}{\underline{\theta}}; \tag{5.2}$$

The highly creative artists can have an informational rent, depending on probabilities of hiring low productivity artists.

Depending on parameters, the optimal probabilities of hiring the artist change. According to those probabilities, there is an information rent for the highly-creative artists; this information rent is due to the distortion in the number of commissioned artworks of the low-creative artist.

6 Some concluding comments

The analysis presented in the previous section highlights that gatekeeping power of declaring some artwork as innovative may create distortions in the selection of artists by galleries. This may happen in particular when innovation affects the price in a substantial way. In this case, a gallery may be attracted by the innovation premium that may charge and pick the wrong (not creative) artist. This circumstance also depends on the characteristics of the gallery, small galleries without sufficient market power and rich collectors among the frequent buyers may be not able to exploit fully the premium that innovation may contribute to determine. Moreover, if payments are delayed, and the interest rate is sufficiently high, a 'small' gallery may be unable to borrow against future sales.

Interestingly, our analysis reveals that a ceiling on the innovation premium, which may concern 'small' galleries, implies that creative artists who have not innovated (for example young talented artists) are more likely to be found in galleries with little market power, because they cannot charge too high prices for innovation. On the contrary, 'superstar' artists who combine creativity with innovation, are always sought after and therefore more likely to end in top galleries. We leave the investigation of a repeated game where innovation may take place in different periods as an agenda for future research.

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Appendix

A Complete information solution

Given that j is selected by the Gallery, the optimal quantity will be solution of the following problem:

$$y_j^* = \arg\max_{y_j} \left[1 + \hat{i}_j + \alpha(\theta_j) - \frac{\beta}{2} y_j\right] y_j - \frac{y_j - \theta_j}{\theta_j} \tag{A.1}$$

The first order condition is:

$$1 + \hat{i}_j + \alpha(\theta_j) - \beta y_j^* - \frac{1}{\theta_j} = 0 \tag{A.2}$$

The solution to the problem is shown in Expression (4.2). Second order condition is satisfied, since $-\beta < 0$. Wages, costs functions and optimal profits are derived by substitution.

Given optimal profits (Expression 4.2), they are maximised in case A, B and C in Table 1 by choosing the Innovator. In case D, hiring the innovator is higher leads to higher profits than choosing the non innovator iff $i \geq \alpha + \frac{1}{\underline{\theta}} - \frac{1}{\overline{\theta}} = \alpha + \Delta$.

B Comparative statics

It is easy to show that:

$$\frac{\Delta y_j^*}{\Delta \theta_j} = y_j^*(\theta_j = \overline{\theta}) - y_j^*(\theta_j = \underline{\theta}) = \frac{1}{\beta}\left[1 + \hat{i}_j + \alpha - \frac{1}{\overline{\theta}}\right] - \frac{1}{\beta}\left[1 + \hat{i}_j - \frac{1}{\underline{\theta}}\right] = \frac{1}{\beta}\left[\alpha + \frac{1}{\underline{\theta}} - \frac{1}{\overline{\theta}}\right] > 0; \tag{B.1}$$

$$\frac{\Delta y_j^*}{\Delta \hat{i}_j} = y_j^*(\hat{i}_j = 1) - y_j^*(\hat{i}_j = 0) = \frac{1}{\beta}\left[1 + i + \alpha(\theta_j) - \frac{1}{\theta_j}\right] - \frac{1}{\beta}\left[1 + \alpha(\theta_j) - \frac{1}{\theta_j}\right] > 0; \tag{B.2}$$

$$\frac{\Delta w_j^*}{\Delta \theta_j} = w_j^*(\theta_j = \overline{\theta}) - w_j^*(\theta_j = \underline{\theta}) = \frac{1}{\beta}\left[\frac{\alpha}{\overline{\theta}} - \left(\frac{1}{\underline{\theta}} - \frac{1}{\overline{\theta}}\right)\left(1 + \hat{i}_j + \frac{1}{\underline{\theta}} + \frac{1}{\overline{\theta}}\right)\right] (>,<,=) 0; \tag{B.3}$$

$$\frac{\Delta w_j^*}{\Delta \hat{i}_j} = w_j^*(\hat{i}_j = 1) - w_j^*(\hat{i}_j = 0) = \frac{1}{\beta \theta_j} > 0; \tag{B.4}$$

$$\frac{\Delta\pi^*}{\Delta\theta_j} = \pi^*(\theta_j = \overline{\theta}) - \pi^*(\theta_j = \underline{\theta}) = \frac{1}{2\beta}\left\{\left[1 + \hat{i}_j + \alpha - \frac{1}{\overline{\overline{\theta}}}\right]^2 - \left[1 + \hat{i}_j - \frac{1}{\underline{\theta}}\right]^2\right\} > 0; \quad \text{(B.5)}$$

$$\frac{\Delta\pi^*}{\Delta\hat{i}_j} = \pi^*(\hat{i}_j = 1) - \pi^*(\hat{i}_j = 0) = \frac{1}{2\beta}\left\{\left[1 + i + \alpha(\theta_j) - \frac{1}{\theta_j}\right]^2 - \left[1 + \alpha(\theta_j) - \frac{1}{\theta_j}\right]^2\right\} > 0; \quad \text{(B.6)}$$

C Maximisation Problem

The Gallery will maximise the expected profits in expression (C.1); each of the four cases (Table 1) represents a combination of the type of artist (low/hight creativity) and of the observable characteristic (innovator/non–innovator).

$$
\begin{aligned}
\pi^e = \quad & kr\left[P_I(\overline{\theta},\overline{\theta})\left(1+i+\alpha-\tfrac{\beta}{2}y_I(\overline{\theta},\overline{\theta})\right)y_I(\overline{\theta},\overline{\theta})+P_{NI}(\overline{\theta},\overline{\theta})\left(1+\alpha-\tfrac{\beta}{2}y_{NI}(\overline{\theta},\overline{\theta})\right)y_{NI}(\overline{\theta},\overline{\theta})\right.\\
& \left.-w_I(\overline{\theta},\overline{\theta})-w_{NI}(\overline{\theta},\overline{\theta})\right]+\\
+ \quad & k(1-r)\left[P_I(\overline{\theta},\underline{\theta})\left(1+i+\alpha-\tfrac{\beta}{2}y_I(\overline{\theta},\underline{\theta})\right)y_I(\overline{\theta},\underline{\theta})+P_{NI}(\overline{\theta},\underline{\theta})\left(1-\tfrac{\beta}{2}y_{NI}(\overline{\theta},\underline{\theta})\right)y_{NI}(\overline{\theta},\underline{\theta})\right.\\
& \left.-w_I(\overline{\theta},\underline{\theta})-w_{NI}(\overline{\theta},\underline{\theta})\right]+\\
+ \quad & (1-k)r\left[P_I(\underline{\theta},\overline{\theta})\left(1+i-\tfrac{\beta}{2}y_I(\underline{\theta},\overline{\theta})\right)y_I(\underline{\theta},\overline{\theta})+P_{NI}(\underline{\theta},\overline{\theta})\left(1+\alpha-\tfrac{\beta}{2}y_{NI}(\underline{\theta},\overline{\theta})\right)y_{NI}(\underline{\theta},\overline{\theta})\right.\\
& \left.-w_I(\underline{\theta},\overline{\theta})-w_{NI}(\underline{\theta},\overline{\theta})\right]+\\
+ \quad & (1-k)(1-r)\left[P_I(\underline{\theta},\underline{\theta})\left(1+i-\tfrac{\beta}{2}y_I(\underline{\theta},\underline{\theta})\right)y_I(\underline{\theta},\underline{\theta})+P_{NI}(\underline{\theta},\underline{\theta})\left(1-\tfrac{\beta}{2}y_{NI}(\underline{\theta},\underline{\theta})\right)y_{NI}(\overline{\theta},\underline{\theta})\right.\\
& -w_I(\underline{\theta},\underline{\theta})-w_{NI}(\underline{\theta},\underline{\theta})]
\end{aligned} \tag{C.1}
$$

Subject to eight participation and incentive compatibility constraints (expressions C.3–C.10), and other eight constraints on probabilities (non–negativity and sum–to–one, C.2).

$$
\begin{aligned}
& P_i(\theta_I,\theta_{NI}) \geq 0; && \theta_I,\theta_{NI}=\{\overline{\theta},\underline{\theta}\} \quad i,j=\{I,NI\};\\
& P_I(\theta_I,\theta_{NI})+P_{NI}(\theta_I,\theta_{NI}) \leq 1; && \theta_I,\theta_{NI}=\{\overline{\theta},\underline{\theta}\}
\end{aligned} \tag{C.2}
$$

Inequalities in C.3–C.6 represent the incentive compatibility constraints for, respectively, the high creativity and innovative artist, for the high productivity but non–innovative artist, and so on. While expressions shown in C.7–C.10 represent the individual rationality constraints for, respectively, the high–creativity and innovative artist, the high creative and non–innovative artist, and so on.

Clearly, PC(L)(I) and IC(H)(I), together, imply PC(H)(I); the same must apply if we consider the non–innovative artist, if PC(L)(NI) and IC(H)(NI) hold true than PC(H)(NI) holds true as well. Furthermore, we can guess that IC(L)(I) and IC(L)(NI) are not binding, then we will check if the guess is correct.

Then, the Gallery will maximise expected profits (C.1) subject to conditions in C.2, and to IC(H)(I), IC(H)(NI), PC(L)(I) and PC(L)(NI) (C.3–C.10).

$$IC(H)(I):$$

$$w_I(\overline{\theta},\overline{\theta})r + w_I(\overline{\theta},\underline{\theta})(1-r) - rP_I(\overline{\theta},\overline{\theta})\frac{y_I(\overline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} - (1-r)P_I(\overline{\theta},\underline{\theta})\frac{y_I(\overline{\theta},\underline{\theta})-\overline{\theta}}{\overline{\theta}} + \tag{C.3}$$

$$-\left[w_I(\underline{\theta},\overline{\theta})r + w_I(\underline{\theta},\underline{\theta})(1-r) - rP_I(\underline{\theta},\overline{\theta})\frac{y_I(\underline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} - (1-r)P_I(\underline{\theta},\underline{\theta})\frac{y_I(\underline{\theta},\underline{\theta})-\overline{\theta}}{\overline{\theta}}\right] \geq 0$$

$$IC(H)(NI):$$

$$w_{NI}(\overline{\theta},\overline{\theta})k + w_{NI}(\underline{\theta},\overline{\theta})(1-k) - kP_{NI}(\overline{\theta},\overline{\theta})\frac{y_{NI}(\overline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} - (1-k)P_{NI}(\underline{\theta},\overline{\theta})\frac{y_{NI}(\underline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} +$$

$$-\left[w_{NI}(\overline{\theta},\underline{\theta})k + w_{NI}(\underline{\theta},\underline{\theta})(1-k) - kP_{NI}(\overline{\theta},\underline{\theta})\frac{y_{NI}(\overline{\theta},\underline{\theta})-\overline{\theta}}{\overline{\theta}} - (1-k)P_{NI}(\underline{\theta},\underline{\theta})\frac{y_{NI}(\underline{\theta},\underline{\theta})-\overline{\theta}}{\overline{\theta}}\right] \geq 0 \tag{C.4}$$

$IC(L)(I):$

$$w_I(\underline{\theta},\overline{\theta})r + w_I(\underline{\theta},\underline{\theta})(1-r) - rP_I(\underline{\theta},\overline{\theta})\frac{y_I(\underline{\theta},\overline{\theta})-\underline{\theta}}{\underline{\theta}} - (1-r)P_I(\underline{\theta},\underline{\theta})\frac{y_I(\underline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}} + \tag{C.5}$$

$$-\left[w_I(\overline{\theta},\overline{\theta})r + w_I(\overline{\theta},\underline{\theta})(1-r) - rP_I(\overline{\theta},\overline{\theta})\frac{y_I(\overline{\theta},\overline{\theta})-\underline{\theta}}{\underline{\theta}} - (1-r)P_I(\overline{\theta},\underline{\theta})\frac{y_I(\overline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}}\right] \geq 0$$

$IC(L)(NI):$

$$w_{NI}(\overline{\theta},\underline{\theta})k + w_{NI}(\underline{\theta},\underline{\theta})(1-k) - kP_{NI}(\overline{\theta},\underline{\theta})\frac{y_{NI}(\overline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}} - (1-k)P_{NI}(\underline{\theta},\underline{\theta})\frac{y_{NI}(\underline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}} +$$

$$-\left[w_{NI}(\overline{\theta},\overline{\theta})k + w_{NI}(\underline{\theta},\overline{\theta})(1-k) - kP_{NI}(\overline{\theta},\overline{\theta})\frac{y_{NI}(\overline{\theta},\overline{\theta})-\underline{\theta}}{\underline{\theta}} - (1-k)P_{NI}(\underline{\theta},\overline{\theta})\frac{y_{NI}(\underline{\theta},\overline{\theta})-\underline{\theta}}{\underline{\theta}}\right] \geq 0 \tag{C.6}$$

$PC(H)(I):$

$$w_I(\overline{\theta},\overline{\theta})r + w_I(\overline{\theta},\underline{\theta})(1-r) - rP_I(\overline{\theta},\overline{\theta})\frac{y_I(\overline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} - (1-r)P_I(\overline{\theta},\underline{\theta})\frac{y_I(\overline{\theta},\underline{\theta})-\overline{\theta}}{\overline{\theta}} \geq 0 \tag{C.7}$$

$PC(H)(NI):$

$$w_{NI}(\overline{\theta},\overline{\theta})k + w_{NI}(\underline{\theta},\overline{\theta})(1-k) - kP_{NI}(\overline{\theta},\overline{\theta})\frac{y_{NI}(\overline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} - (1-k)P_{NI}(\underline{\theta},\overline{\theta})\frac{y_{NI}(\underline{\theta},\overline{\theta})-\overline{\theta}}{\overline{\theta}} \geq 0 \tag{C.8}$$

$PC(L)(I):$

$$w_I(\underline{\theta},\overline{\theta})r + w_I(\underline{\theta},\underline{\theta})(1-r) - rP_I(\underline{\theta},\overline{\theta})\frac{y_I(\underline{\theta},\overline{\theta})-\underline{\theta}}{\underline{\theta}} - (1-r)P_I(\underline{\theta},\underline{\theta})\frac{y_I(\underline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}} \geq 0 \tag{C.9}$$

$PC(L)(NI):$

$$w_{NI}(\overline{\theta},\underline{\theta})k + w_{NI}(\underline{\theta},\underline{\theta})(1-k) - kP_{NI}(\overline{\theta},\underline{\theta})\frac{y_{NI}(\overline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}} - (1-k)P_{NI}(\underline{\theta},\underline{\theta})\frac{y_{NI}(\underline{\theta},\underline{\theta})-\underline{\theta}}{\underline{\theta}} \geq 0 \tag{C.10}$$

Note that the objective function is quadratic in the tasks assigned and linear in the probabilities and wages. The maximisation problem is:

$$\max \pi^e + \lambda_1 [IC(H)(I)] + \lambda_2 [IC(H)(NI)] + \lambda_3 [PC(L)(I)] + \lambda_4 [PC(L)(NI)] \tag{C.11}$$

F.O.Cs are:

$$\lambda_1 = k\overline{\theta}\left[1 + i + \alpha - \beta y_I(\overline{\theta},\overline{\theta})\right] = k\overline{\theta}\left[1 + i + \alpha - \beta y_I(\overline{\theta},\underline{\theta})\right] > 0 \tag{C.12}$$

$$\lambda_3 = \frac{\lambda_1 \theta}{\overline{\theta}} + (1-k)\underline{\theta}\left[1 + i - \beta y_I(\underline{\theta},\overline{\theta})\right] = \frac{\lambda_1 \theta}{\overline{\theta}} + (1-k)\underline{\theta}\left[1 + i - \beta y_I(\underline{\theta},\underline{\theta})\right] > 0 \tag{C.13}$$

$$\lambda_2 = r\overline{\theta}\left[1 + \alpha - \beta y_{NI}(\overline{\theta},\overline{\theta})\right] = r\overline{\theta}\left[1 + \alpha - \beta y_{NI}(\underline{\theta},\overline{\theta})\right] > 0 \tag{C.14}$$

$$\lambda_4 = \frac{\lambda_2 \theta}{\overline{\theta}} + (1-r)\underline{\theta}\left[1 - \beta y_{NI}(\overline{\theta},\underline{\theta})\right] = \frac{\lambda_2 \theta}{\overline{\theta}} + (1-r)\underline{\theta}\left[1 - \beta y_{NI}(\underline{\theta},\underline{\theta})\right] > 0 \tag{C.15}$$

Therefore, the four constraints hold with equality (because multipliers are positive); It is also the case that:

$$y_I(\overline{\theta},\overline{\theta}) = y_I(\overline{\theta},\underline{\theta}) = y_I(\overline{\theta}); \qquad y_I(\underline{\theta},\overline{\theta}) = y_I(\underline{\theta},\underline{\theta}) = y_I(\underline{\theta});$$
$$y_{NI}(\overline{\theta},\overline{\theta}) = y_{NI}(\underline{\theta},\overline{\theta}) = y_{NI}(\overline{\theta}); \quad y_{NI}(\overline{\theta},\underline{\theta}) = y_{NI}(\underline{\theta},\underline{\theta}) = y_{NI}(\underline{\theta}); \tag{C.16}$$

By substituting the constraints (which hold with equality), and considering the above relationships about tasks, we can rearrange the problem in Expression C.17 as:

$$\begin{aligned}
\max \quad & k\left\{A\left[\left(1+i+\alpha-\tfrac{\beta}{2}y_I(\overline{\theta})\right)y_I(\overline{\theta})-\tfrac{y_I(\overline{\theta})-\overline{\theta}}{\overline{\theta}}\right]-By_I(\underline{\theta})\Delta\right\}\\
& +(1-k)\left\{B\left[\left(1+i-\tfrac{\beta}{2}y_I(\underline{\theta})\right)y_I(\underline{\theta})-\tfrac{y_I(\underline{\theta})-\underline{\theta}}{\underline{\theta}}\right]\right\}\\
& +r\left\{C\left[\left(1+\alpha-\tfrac{\beta}{2}y_{NI}(\overline{\theta})\right)y_{NI}(\overline{\theta})-\tfrac{y_{NI}(\overline{\theta})-\overline{\theta}}{\overline{\theta}}\right]-Dy_{NI}(\underline{\theta})\Delta\right\}\\
& +(1-r)\left\{D\left[\left(1-\tfrac{\beta}{2}y_{NI}(\underline{\theta})\right)y_{NI}(\underline{\theta})-\tfrac{y_{NI}(\underline{\theta})-\underline{\theta}}{\underline{\theta}}\right]\right\}
\end{aligned} \tag{C.17}$$

Where:

$$\begin{aligned}
A = \;& rP_I(\overline{\theta},\overline{\theta}) + (1-r)P_I(\overline{\theta},\underline{\theta}); \quad B = \; rP_I(\underline{\theta},\overline{\theta}) + (1-r)P_I(\underline{\theta},\underline{\theta});\\
C = \;& kP_I(\overline{\theta},\overline{\theta}) + (1-k)P_I(\underline{\theta},\overline{\theta}); \quad D = \; kP_I(\overline{\theta},\underline{\theta}) + (1-r)P_I(\underline{\theta},\underline{\theta});
\end{aligned} \tag{C.18}$$

Differentiating for the tasks we have that:

$$y_I(\overline{\theta}) = \tfrac{1}{\beta}\left[1+i+\alpha-\tfrac{1}{\overline{\overline{\theta}}}\right];\quad y_I(\underline{\theta}) = \tfrac{1}{\beta}\left[1+i-\tfrac{1}{\underline{\theta}}-\tfrac{k}{1-k}\Delta\right];$$

$$y_{NI}(\overline{\theta}) = \tfrac{1}{\beta}\left[1+\alpha-\tfrac{1}{\overline{\overline{\theta}}}\right];\qquad y_{NI}(\underline{\theta}) = \tfrac{1}{\beta}\left[1-\tfrac{1}{\underline{\theta}}-\tfrac{r}{1-r}\Delta\right]; \tag{C.19}$$

The quantity of art for high creativity artists is equal to that in first best. For the low skilled artist, the optimal quantity is smaller than that in first best. By substituting the optimal quantities in Expression C.17 we can compute the optimal probabilities and wages. Since the problem is linear in probabilities, we should have that each probability is either one or zero: $P_I(\theta_I, \theta_{NI}) = 1 - P_{NI}(\theta_I, \theta_{NI})$. So:

$$\begin{aligned}
\pi^e = \;& 1 + Ak\underbrace{\left[\frac{1}{2\beta}\left(1+i+\alpha-\frac{1}{\overline{\overline{\theta}}}\right)^2\right]}_{u_1} + B(1-k)\underbrace{\left[\frac{1}{2\beta}\left(1+i-\frac{1}{\underline{\theta}}-\frac{k}{1-k}\Delta\right)^2\right]}_{u_2} \\
+ \;& Cr\underbrace{\left[\frac{1}{2\beta}\left(1+\alpha-\frac{1}{\overline{\overline{\theta}}}\right)^2\right]}_{u_3} + D(1-r)\underbrace{\left[\frac{1}{2\beta}\left(1-\frac{1}{\underline{\theta}}-\frac{r}{1-r}\Delta\right)^2\right]}_{u_4} \\
= \;& 1 + ru_3 + (1-r)u_4 + rkP_I(\overline{\theta},\overline{\theta})\left[u_1-u_3\right] + k(1-r)P_I(\overline{\theta},\underline{\theta})\left[u_1-u_4\right] \\
+ \;& r(1-k)P_I(\underline{\theta},\overline{\theta})\left[u_2-u_3\right] + (1-k)(1-r)P_I(\underline{\theta},\underline{\theta})\left[u_2-u_4\right]
\end{aligned} \tag{C.20}$$

The maximisation problem, now, is a linear programming problem. Since u_1 is higher than u_2, u_3 and u_4, the expected profit is maximised by setting $P_I(\overline{\theta},\overline{\theta}) = P_I(\overline{\theta},\underline{\theta}) = 1$, always. The other results depend on the values of u_2, u_3 and u_4.

When $u_2 > u_3$ then $P_I(\underline{\theta},\overline{\theta}) = 1$, and zero otherwise. Moreover, when $u_2 > u_4$ then $P_I(\underline{\theta},\underline{\theta}) = 1$, and zero otherwise.

It is also worth to show that $u_2 < u_4$ implies $u_2 < u_3$. This because $u_2 < u_4 \iff i < \frac{k-r}{(1-k)(1-r)}\Delta$ and $u_2 < u_3 \iff i < \alpha + \frac{1}{(1-k)}\Delta$ and $\frac{k-r}{(1-k)(1-r)} < \frac{1}{(1-k)}$. Thus there are three possible solutions (Table 4).

$P_I(\overline{\theta},\overline{\theta})$	1	1	1
$P_I(\overline{\theta},\underline{\theta})$	1	1	1
$P_I(\underline{\theta},\underline{\theta})$	0	1	1
$P_I(\underline{\theta},\overline{\theta})$	0	0	1
	$i < \frac{k-r}{(1-k)(1-r)}\Delta$	$\frac{k-r}{(1-k)(1-r)}\Delta < i < \alpha + \frac{1}{(1-k)}\Delta$	$i > \alpha + \frac{1}{(1-k)}\Delta$
	$u_2 < u_4$	$u_2 < u_3$	$u_2 > u_3$
	$u_2 < u_3$	$u_2 > u_4$	$u_2 > u_4$

Table 4: Three Solutions with respect to i

Finally wages are such that:

$$w_I(\overline{\theta},\overline{\theta}) = \frac{y_I(\overline{\theta})-\overline{\theta}}{\overline{\theta}} + P_I(\underline{\theta},\overline{\theta})y_I(\underline{\theta})\Delta; \qquad w_I(\overline{\theta},\underline{\theta}) = \frac{y_I(\overline{\theta})-\overline{\theta}}{\overline{\theta}} + P_I(\underline{\theta},\underline{\theta})y_I(\underline{\theta})\Delta;$$

$$w_I(\underline{\theta},\overline{\theta}) = P_I(\underline{\theta},\overline{\theta})\frac{y_I(\underline{\theta})-\underline{\theta}}{\underline{\theta}}; \qquad w_I(\underline{\theta},\underline{\theta}) = P_I(\underline{\theta},\underline{\theta})\frac{y_I(\underline{\theta})-\underline{\theta}}{\underline{\theta}};$$

$$w_{NI}(\overline{\theta},\overline{\theta}) = 0; \qquad w_{NI}(\overline{\theta},\underline{\theta}) = 0;$$

$$w_{NI}(\underline{\theta},\overline{\theta}) = P_{NI}(\underline{\theta},\overline{\theta})\frac{y_{NI}(\overline{\theta})-\overline{\theta}}{\overline{\theta}} + P_{NI}(\underline{\theta},\underline{\theta})y_{NI}(\underline{\theta})\Delta; \qquad w_{NI}(\underline{\theta},\underline{\theta}) = P_{NI}(\underline{\theta},\underline{\theta})\frac{y_{NI}(\underline{\theta})-\underline{\theta}}{\underline{\theta}}; \tag{C.21}$$

The other constraints – not considered in the maximisation problem – are satisfied; thus, our guess is true.